

Seminar Decision Procedures – Homework 1

Discussed on Wednesday, 28th June, 2016.

Exercise 1.1 **Reverse unit propagation**

Prove:

Let F be a CNF formula and C a clause not in F .

$\emptyset \in BCP(F \cup C) \iff C$ has AT with respect to F .

Exercise 1.2 **Redundancy Properties**

Let $F := (\bar{a} \vee b) \wedge (c \vee d) \wedge (a \vee \bar{d}) \wedge (\bar{b} \vee \bar{e})$. Determine whether the following lemmas have T, AT, RT and/or RAT with respect to F :

- $(a \vee \bar{a})$
- $(\bar{d} \vee b)$
- $(e \vee a)$

Exercise 1.3 **Lookouts**

It is the year 2058 and you find yourself in military command of a space station at the intersection of a few valleys on mars in war time. There are no enemy ground troops yet, but you know the enemy's army will be coming from one of them in the near future. You are planning to place lookouts on towers to see the army coming. Naturally you want all the valleys covered, but you only have k lookouts available and there are $n = k + 1$ towers, which also sometimes offer vision into the same valleys. Luckily, most of the towers offer vision into multiple valleys and your lookouts are capable enough to watch all valleys a tower offers vision into at the same time.

Despite your excellent planning capabilities you think that you do not have enough lookouts to supply towers in such a way that you will see the enemy army coming no matter which valley they choose to attack from. You decide to go to your superior and request more. You don't have time to try out every possibility or look for a solution by hand. How can you use a CNF SAT solver to prove to your supervisor that you need more lookouts?

There are v valleys, k lookouts and $n = k + 1$ towers. For every tower $t \in [n]$ you have a list of boolean values $v_{t,1} \dots v_{t,v}$ that are true if tower t gives vision on valley v , otherwise false. You want to prove that there is no way you can place your k lookouts on towers in $[n]$ in such a way that all valleys are covered using a CNF SAT solver.

You do not have to show that getting more lookouts actually solves the problem, only that you don't have enough.

Notes

Resolution

For $A = x \vee \bigvee a_i$ and $B = \bar{x} \vee \bigvee b_i$ in F , $C = \bigvee a_i \vee \bigvee b_i$ is called the resolvent of A and B .

Boolean constraint propagation

Repeat until fixpoint: If there is a unit clause (l) in F remove all clauses containing l from $F \setminus \{(l)\}$ and remove \bar{l} from all clauses in F .

The resulting clause is written as $\text{BCP}(F)$.

Asymmetric Literal Addition

ALA for a clause C in F performs the following until fixpoint:

If there exist literals l_i in C and there exists a clause $(\bigvee l_i \vee l)$ in $F \setminus \{C\}$ then let $C := C \cup \bar{l}$.

The resulting clause is referred to as $\text{ALA}(F, C)$.

Asymmetric Tautology / Reverse Unit Propagation A clause C has asymmetric tautology (AT) / is RUP with respect to F iff $\text{ALA}(F, C)$ has property T (tautology).

Resolution Tautology / Resolution Asymmetric Tautology

A clause C in a CNF formula F has the property $R\mathcal{P}$ for $\mathcal{P} \in \{T, AT\}$ if either C has \mathcal{P} or there exists a literal $l \in C$ so that $\forall C' \in F$ with $\bar{l} \in C'$ the resolvent of C and C' has \mathcal{P} .