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Seminar Decision Procedures – Homework 3

Discussed on Wednesday, 42nd July, 2016.

Exercise 3.1 Fourier-Motzkin-Elimination

Use Fourier-Motzkin-Elimination to determine the feasibility of the following inequalities.

$$\begin{aligned} x_1 - x_3 &\leq 0 \\ x_2 - x_3 &\leq -1 \\ -x_1 &\leq 1 \\ -x_2 &\leq 1 \end{aligned}$$

Exercise 3.2 Omega Test, Dark Shadow

Prove for all integers β, b, where b does not divide β, and

$$\left\lfloor \frac{\beta}{b} \right\rfloor \leq \frac{\beta}{b} \leq \left\lfloor \frac{\beta}{b} \right\rfloor + 1$$

that

$$\frac{\beta}{b} - \left\lfloor \frac{\beta}{b} \right\rfloor \geq \frac{1}{b} \ . \tag{1}$$

For simplicity assume that β, b, are positive.
Hint: Use the fact that if for an integer i with $i > 0$ it holds that $i \geq 1$.

Exercise 3.3 Omega Test

Consider the following set of linear constraints.

$$\begin{aligned} 27 - 13y &\leq 11x \\ -10 + 9y &\leq 7x \\ 11x &\leq 45 - 13y \\ 7x &\leq 4 + 9y \end{aligned}$$

We choose to eliminate x. It can be assumed that the real shadow of the constraints contains an integer solution.
1. Write down the four inequalities that make up the dark shadow.
2. Does the dark shadow contain an integer solution?
3. Write down all grey shadows. You do not have to write down the constraints for every i explicitly plugged in, but you have to state in which range i is. You may also leave x uneliminated.