

Algorithms for the Undecided – Homework 4

Discussed on Tuesday, 5th July, 2016.

Exercise 4.1 Story

Pizza baker Donald is in a pickle. His oven has caught fire and all the beautiful ingredients in his kitchen will be destroyed in the ensuing fire. His plan is to use the additional heat to abbreviate the cooking time. To do so, he gets 3 of his employees to join him. They got a deal, that promises each of them 10 pizzas of all the ones made.

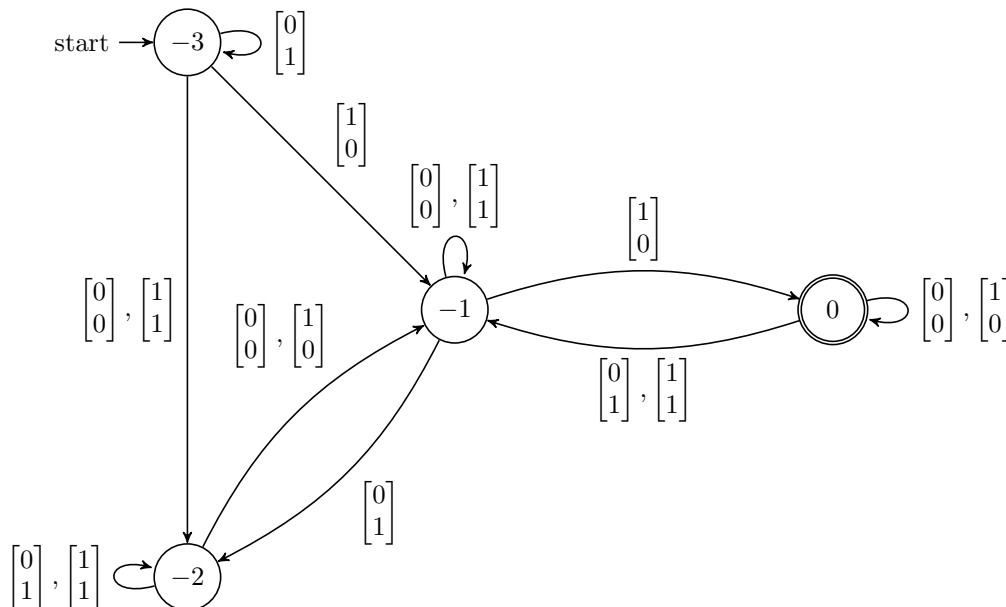
Tomatoes	Cheese	Basil	Oil	Garlic	Oregano	Anchovies	Capers	Mushrooms	Parsley
120	1960	144	700	16	180	144	112	88	96

Pizza	Toppings
Margherita	2 Tomatoes, 60 Cheese, 8 Basil, 10 Oil
Marinara	2 Tomatoes, 1 Garlic, 6 Oregano, 10 Oil
Napoletana	2 Tomatoes, 6 Anchovies, 6 Oregano, 8 Capers, 10 Oil
Fungi	2 Tomatoes, 40 Cheese, 4 Mushrooms, 8 Parsley, 10 Oil
Boscaiola	40 Cheese, 4 Mushrooms, 6 Anchovies, 10 Oil

Donald just wants to risk the four lives, if he can use up all of the ingredients and he can keep more pizzas to himself, then he has to give away. Give a formula in Presburger arithmetic that encodes this problem.

Exercise 4.2 Deconstruct

Find a Presburger formula φ that is represented by the DFA $A_\varphi = (Q, \Sigma, \delta, q_0, F)$ and provide a satisfying assignment for the free variables in decimal notation.



Exercise 4.3 Construct

Construct automata for the formulas and provide a satisfying assignment for the free variables in decimal notation. The last one is a bonus task and will not be graded, there won't be a solution either.

$$\varphi_1 := x_1 - 2x_2 \leq 3, \varphi_2 := -2x_1 - x_2 \leq -4, \varphi := \varphi_1 \vee \varphi_2$$