

Exercise Sheet – Combination of Theories

Exercise 1

Prove that the following combination of Linear Arithmetic over \mathbb{R} and Uninterpreted Functions is satisfiable using the Nelson–Oppen procedure:

$$(x_2 \geq 0) \wedge (f(x_1) = x_3) \wedge (f(x_1) \geq x_1) \wedge (x_1 - x_2 \geq x_3) \wedge (f(x_3) \geq 0)$$

- a) Show the formula after purification with optimization, where the same functions or values are assigned to the same auxiliary variable.

- b) Show the equalities implied by the purified formula and follow the rest of the Nelson–Oppen procedure until no further equality can be implied.

Exercise 2

Prove that the following combination of three different theories (Linear Arithmetic over \mathbb{R} , Uninterpreted Functions starting with the letters a-m and Uninterpreted Functions starting with the letters n-z) is unsatisfiable using the Nelson–Oppen procedure. Therefore, use a table with three columns (F_1, F_2, F_3), one per theory.

$$(x_2 \geq x_1) \wedge (x_1 - x_3 \geq x_2) \wedge (z(f(x_1) - f(x_2)) \neq z(x_3)) \wedge (x_3 \geq 0)$$

Exercise 3

Prove that the following combination of Linear Arithmetic over \mathbb{Z} and Uninterpreted Function is unsatisfiable using the Nelson–Oppen procedure for non-convex formulas.

$$1 \leq x \wedge x \leq 2 \wedge f(x) \neq f(1) \wedge f(x) \neq f(2)$$

Exercise 4

Prove that the following combination of Linear Arithmetic over \mathbb{Z} and Uninterpreted Function is unsatisfiable using the Nelson–Oppen procedure for non-convex formulas.

$$(1 \leq x_1) \wedge (x_2 \leq 3) \wedge (x_3 \leq x_2) \wedge (x_1 \leq 3 - x_3) \wedge (x_3 = 1) \wedge g(x_1) \wedge \neg g(2) \wedge f(x_2) \neq f(x_1) \wedge f(x_2) \neq f(3) \wedge \neg g(x_2 - x_1)$$