

Goals First steps in Simpl and concepts and rules of semantics and Hoare logic.

For the non-Isabelle exercises on this sheet, please answer each question in a complete sentence.

Exercise 1 [3] Basic notions

1. What is the idea of a *shallow embedding*?
2. Name some part of Simpl which is embedded shallowly and some part which is embedded deeply.
3. The *Soundness* of the Hoare logic for Simpl is expressed by the following theorem. Explain this proposition in a sentence.

$$\Gamma, \Theta \vdash P \ c \ Q, A \implies \Gamma, \Theta \models P \ c \ Q, A$$

Exercise 2 [4] Semantics

1. Explain how the following rules model a conditional expression:

$$\begin{aligned} \llbracket s \in b; \Gamma \vdash \langle c_1, \text{Normal } s \rangle \Rightarrow t \rrbracket &\implies \Gamma \vdash \langle \text{Cond } b \ c_1 \ c_2, \text{Normal } s \rangle \Rightarrow t \\ \llbracket s \notin b; \Gamma \vdash \langle c_2, \text{Normal } s \rangle \Rightarrow t \rrbracket &\implies \Gamma \vdash \langle \text{Cond } b \ c_1 \ c_2, \text{Normal } s \rangle \Rightarrow t \end{aligned}$$

2. The Basic command encodes state updates. Explain the rule:

$$\Gamma \vdash \langle \text{Basic } f, \text{Normal } s \rangle \Rightarrow \text{Normal } (fs)$$

3. How do the rules above rely on a shallow embedding-approach?
4. The Simpl state type is:

`datatype xstate = Normal state | Abrupt state | Fault | Stuck`

How can the state Abrupt ever be reached? What is the meaning of this state?

Exercise 3 [3] Hoare Logic

1. The Hoare logic provides the following rule for state updates. Why does it match the semantic given above?

$$\Gamma, \Theta \vdash \llbracket s. fs \in Q \rrbracket (\text{Basic } f) \ Q, A$$

2. For the branches of a conditional statement, one wants to have the test result as additional knowledge. How is this realized in each of the following rules?

$$\llbracket \Gamma, \Theta \vdash (P \wedge b) \ c_1 \ Q, A; \Gamma, \Theta \vdash (P \wedge \neg b) \ c_2 \ Q, A \rrbracket \implies \Gamma, \Theta \vdash P \ (\text{Cond } b \ c_1 \ c_2) \ Q, A$$

$$\Gamma, \Theta \vdash (P \wedge b) \ c \ P, A; \implies \Gamma, \Theta \vdash P \ (\text{While } b \ c) \ (P \wedge \neg b), A$$

3. The classic rule for While loops is stated below. Why is the usual name *invariant* for I a sensible choice?

$$\llbracket P \subseteq I; \Gamma, \Theta \vdash (I \wedge b) \ c \ I, A; I \wedge \neg b \subseteq Q \rrbracket \implies \Gamma, \Theta \vdash P \ (\text{While } b \ c) \ Q$$

Exercise 4 [3] Getting Started with Simpl

For this and later exercises you need the Simpl package from the *Archive of Formal Proofs*. See the `Sheet04.thy` for the Simpl exercises.