

Goals Manipulate lists on the heap and use ghost variables.

Exercise 1 [3] Prepend a List Node

The cons operation (insert a new element at the beginning of a list) is easy to implement on the heap. One just needs to set the `next`-pointer of the new node to the begin of the existing list. Moreover, the old list stays unchanged, which makes this operation so useful for functional languages.

```
pre: "list node-alloc node-next p XS NULL  $\wedge$  node-alloc q"
post: "list node-alloc node-next q (q # XS) NULL  $\wedge$ 
      list node-alloc node-next p XS NULL"
{ *
  q  $\rightarrow$  next = p;
* }
```

Unfortunately, the given pre-condition is too weak. Insert the necessary additional assumptions and prove the correctness. (Have a look at the “Cheat Sheet” from the lecture).

Exercise 2 [4] Test for an Element

The following program tests whether the value v is contained in the list p . Prove the program correct. For this, annotate the program with ghost variables, so you do not have to provide witnesses for existential quantifiers manually.

```
pre: "list node-alloc node-next p XS NULL  $\wedge$  DS = list-data node-data XS"
post: "r  $\neq$  0  $\leftrightarrow$  v  $\in$  set DS"
{ *
  r = 0;
  /*@ True */
  while (p != null && r == 0) {
    if (p  $\rightarrow$  data == v) {
      r = 1;
    }
    p = p  $\rightarrow$  next;
  }
* }
```

Exercise 3 [5] Insert an Element (Hard)

Verify the following code which inserts a node q at the position pos of the list p and returns the modified list in r .

```
if (pos == 0) {
  q  $\rightarrow$  next = p;
  r = q;
} else {
  r = p;
  prev = p;
  p = p  $\rightarrow$  next;
  pos = pos - 1;
  while (0 < pos) {
    prev = p;
    p = p  $\rightarrow$  next;
  }
}
```

```
    pos = pos - 1;
}
prev->next = q;
q->next = p;
}
```