

Interactive Software Verification

Spring Term 2013

Holger Gast

`gasth@in.tum.de`

9.7.2013

- Motivation: Short-comings of Burstall's memory model
 - No byte-addressed memory (low-level C programs)
 - Disjointness assertions cumbersome
 - Disjointness proofs time-consuming
- Separation Logic
 - Express heap layout syntactically
 - ⇒ Aliasing/disjointness clear from syntax
 - Manipulate heap formulae algebraically
- So far: basic notions & definitions

- Symbolic Execution — the key to success [2, 11, 4, 5]

⇒ Develop framework within Isabelle

- Re-use Simpl-embedding [6]

Recap

- Keep memory in global variable `heap`

datatype `addr` = `Addr nat`

datatype `heap` = `Heap "addr \rightarrow heap-val"`

- Establish heap access layer

`heap-get ct p h` \equiv `ty-val ct (map (the \circ hcnt h) (addr-ivl p (ty-size ct)))`

`heap-put ct p v h` \equiv

`Heap (hcnt h ++ (λ q. if $q \in$ set (addr-ivl p (ty-size ct))
 then Some (nth (ty-rep ct v) (q \ominus p))
 else None))`

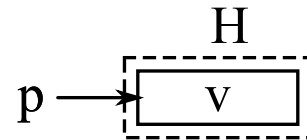
`heap-acc-cond ct p` \equiv

`λ h. set (addr-ivl p (ty-size ct)) \subseteq hdom h`

- Parameterized by **type descriptors**: representation of values

Recap: Spatial Assertions

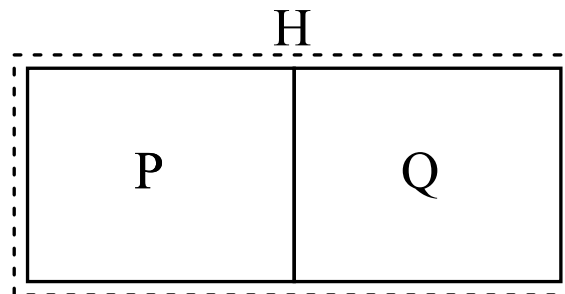
- Points-to: values in the heap



$p \mapsto \text{ct}. v \equiv$

$\lambda h. \text{hdom } h = \text{set}(\text{addr-ivl } p (\text{ty-size ct})) \wedge \text{heap-get ct } p \text{ } h = v \wedge p \neq \text{NULL}$

- Star: disjoint parts of the heap



$P \star Q \equiv \lambda h0. \exists h h'. h \perp h' \wedge h0 = h \uplus h' \wedge P h \wedge Q h'$

- Core idea: tight specifications

- C Structs

- define-struct**

- "struct node { int data; struct node *next; }"

- HOL value is an Isabelle record

- $(\lambda \text{node-data} = \text{data}, \text{node-next} = \text{next})$

- Introduce constants for field offsets (fo-...)

- Define spatial abstraction predicate

- $$\text{node } p \ v \equiv \mathcal{E} \ \text{data} \ \text{next}. \ p \oplus \ \text{fo-node-data} \mapsto \text{cty-int. data} \star$$

$$p \oplus \ \text{fo-node-next} \mapsto \text{cty-ptr. next} \cdot$$

$$v = (\lambda \text{node-data} = \text{data}, \text{node-next} = \text{next})$$

\Rightarrow Simple notation for records on the heap

Symbolic Execution

- Goal: automatic verification in separation logic [2, 11, 4, 5]
- Hoare rule for forward reasoning

$$\{ p \mapsto - \star Q \} * p := e \{ p \mapsto e \star Q \}$$

- Procedural interpretation
 - Re-arrange pre-condition by syntactic manipulations
 - Match against pre-condition of rule
- ⇒ Fills Q
 - Replace Q and e in post-condition
- ⇒ Compute post-condition from pre-condition
 - Local variables: treat as in Simpl

- General manipulation of sep-logic formulae hard [8, 10]
- Specifications of most programs fall into fragment

$$\exists x_1 \dots x_n. P_1 \star \dots \star P_m \wedge (Q_1 \wedge \dots \wedge Q_r)$$

with spatial assertions P_i and pure assertions Q_j

- Intuition
 - spatial part used to “read” values from heap
 - pure part specifies relationships between values

⇒ Term: **symbolic heap**

- In Isabelle (because other symbols are taken)

$$\mathcal{E}x_1 \dots x_n. P_1 \star \dots \star P_m \cdot Q_1 \wedge \dots \wedge Q_r$$

- As premise: $\bigwedge x_1 \dots x_n. \llbracket (P_1 \star \dots \star P_m) \text{ heap}; Q_1; \dots Q_r \rrbracket \implies \dots$

- Consider example

```
*p = 1;
```

```
r = *q;
```

```
*q = 2;
```

- Simpl generates

$$\begin{aligned} & (p \mapsto \text{cty-int. } X \star q \mapsto \text{cty-int. } Y) \text{ heap} \implies \\ & \text{heap-acc-cond cty-int } p \text{ heap} \wedge \\ & \text{heap-acc-cond cty-int } q \text{ (heap-put cty-int } p \text{ 1 heap)} \wedge \\ & (p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } 2 \cdot \\ & \text{heap-get cty-int } q \text{ (heap-put cty-int } p \text{ 1 heap)} = Y) \\ & (\text{heap-put cty-int } q \text{ 2 (heap-put cty-int } p \text{ 1 heap)}) \end{aligned}$$

\Rightarrow Where is the “execution” in this mess?

- Recall side-effects: $a[i] = 42$; with post: $\text{aget } a \ i = 42$
 $\llbracket 0 \leq i; i < \text{length } a \rrbracket \implies \text{acheck } i \ a \wedge \mathbf{\text{aget } (aset \ i \ 42 \ a) \ i = 42}$
 - Analogous: $*p = 1$; $r = *q$; $*q = 2$;
 $(p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } 2 \cdot$
 $\text{heap-get } \text{cty-int } q \ (\mathbf{\text{heap-put } \text{cty-int } p \ 1 \ \text{heap}}) = Y)$
 $\mathbf{(\text{heap-put } \text{cty-int } q \ 2 \ (\text{heap-put } \text{cty-int } p \ 1 \ \text{heap}))}$
- \Rightarrow Obtain assertion about “changed heap”, as usual
- Central insight [6]
 The nesting of the heap access conditions reflects the original execution order.

- Consider again `*p = 1; *r = *q;`
`heap-get cty-int q (heap-put cty-int p 1 heap)`
- Plan of method `prep_vc` [6]
 - Name all heap accesses (get & put)
 - ⇒ Have names for all intermediate heap states
 - Build dependency graph
 - Nested accesses executed before outer accesses
 - `heap-put` after `heap-get` on same heap state
 - Introduce let-bindings according to dependencies
- ⇒ Let-binding reflect execution order [1, 7, 9]
- Technical detail: use `HeapOp ≡ Let` as markup

- Code $*p = 1; r = *q; *q = 2;$
- Result

```

heap-acc-cond cty-int p heap  $\wedge$ 
  (let H-1 = heap-put cty-int p 1 heap
   in heap-acc-cond cty-int q H-1  $\wedge$ 
    (let H-2 = heap-get cty-int q H-1;
     H-3 = heap-put cty-int q 2 H-1
     in (p  $\mapsto$  cty-int. 1  $\star$  q  $\mapsto$  cty-int. 2  $\cdot$  H-2 = Y) H-3))

```

- Remark: technically

```

heap-acc-cond cty-int p heap  $\wedge$ 
  (heapop H-1  $\leftarrow$  heap-put cty-int p 1 heap;
   heap-acc-cond cty-int q H-1  $\wedge$ 
  (heapop H-2  $\leftarrow$  heap-get cty-int q H-1;
   heapop H-3  $\leftarrow$  heap-put cty-int q 2 H-1;
   (p  $\mapsto$  cty-int. 1  $\star$  q  $\mapsto$  cty-int. 2  $\cdot$  H-2 = Y) H-3))

```

- Idea: current state reflected in goal's premises
- Example

$$\begin{aligned}
 & (p \mapsto \text{cty-int. } X \star q \mapsto \text{cty-int. } Y) \text{ heap} \implies \\
 & \text{heap-acc-cond cty-int } p \text{ heap} \wedge \\
 & (\text{heapop } H-1 \leftarrow \mathbf{\text{heap-put cty-int } p \ 1} \text{ heap;} \\
 & \text{heap-acc-cond cty-int } q \ H-1 \wedge
 \end{aligned}$$

- Next step

$$\begin{aligned}
 & (p \mapsto \mathbf{\text{cty-int. } 1} \star q \mapsto \text{cty-int. } Y) h \implies \\
 & \text{heap-acc-cond cty-int } q \ h \wedge \\
 & (\text{heapop } H-2 \leftarrow \mathbf{\text{heap-get cty-int } q} \ h; \\
 & \text{heapop } H-3 \leftarrow \text{heap-put cty-int } q \ 2 \ h; \\
 & (p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } 2 \cdot H-2 = Y) H-3)
 \end{aligned}$$

- Next

$$\begin{aligned}
 & (q \mapsto \text{cty-int. } Y \star p \mapsto \text{cty-int. } 1) h \implies \\
 & \text{HeapOp}(\mathbf{\text{heap-put cty-int } q \ 2} \ h) \\
 & (p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } 2 \cdot \mathbf{Y} = Y)
 \end{aligned}$$

- Recall classical rule

$$\{ p \mapsto - \star Q \} * p := e \{ p \mapsto e \star Q \}$$

- In the case of goals: replace heap premise with different one

$$\begin{array}{l} \llbracket (p \mapsto \text{ct. } a \star H) h; \\ \text{ty-ok ct;} \\ \wedge h. (p \mapsto \text{ct. } v \star H) h \implies Q h \\ \rrbracket \implies \text{HeapOp}(\text{heap-put ct } p \ v \ h) \ Q \end{array}$$

- Match first premise against current heap
 - Side-condition: type is well-formed
 - Result: new heap h with new assertion
 - Auxiliary tactic: drop old heap assertion
- \Rightarrow Effect: “in-place” update of heap assertion in current goal

- Situation

$$(p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } Y) h \implies \\ (\text{heapop-H-2} \leftarrow \text{heap-get cty-int } q h; \dots)$$

- Basic insight: lookup “the value” at p

$$(p \mapsto \text{ct. } v \star R) h \implies \text{heap-get ct } p h = v$$

- Yields

$$(p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } Y) h \implies \\ (\text{heapop-H-2} \leftarrow Y \dots)$$

- Then unfold the let-binding \Leftrightarrow syntactic replacement

- Situation

$$(p \mapsto \text{cty-int. } X \star q \mapsto \text{cty-int. } Y) \text{ heap} \implies \\ \text{heap-acc-cond ct-int } p \text{ heap} \wedge \dots$$

- Rule: points-to includes side-condition on allocatedness

$$(p \mapsto \text{ct. } a \star H) h \implies \text{heap-acc-cond ct } p h$$

\Rightarrow Simply apply rule to prove side-condition

- All three rules have premise $p \mapsto \text{ct. } a \star H$
- Re-arrange compartments of current heap to match
- Star \star is associative & commutative \Leftrightarrow order irrelevant
- Likewise: heap-matching in final proof obligation

- In example:

$$\begin{aligned} & (q \mapsto \text{cty-int. } 2 \star p \mapsto \text{cty-int. } 1) \text{ ha} \implies \\ & (p \mapsto \text{cty-int. } 1 \star q \mapsto \text{cty-int. } 2) \text{ ha} \end{aligned}$$

\Rightarrow Method heap

- Next up: sub-structures & unfolding

define-struct

"struct node { int data; struct node *next; }"

- Example: $r = p \rightarrow \text{data}$;

node p v heap \implies

heap-acc-cond cty-int (**p \oplus fo-node-data**) heap \wedge
 node p v heap \wedge
 (heapop H-1 \leftarrow heap-get cty-int (p \oplus fo-node-data) heap;
 H-1 = node-data v)

- Heap matching applies unfolding rules automatically [3, 2]
- Example

node p v =

(\mathcal{E} data next.

p \oplus fo-node-data \mapsto cty-int. data \star p \oplus fo-node-next \mapsto cty-ptr. next \cdot
 v = (\downarrow node-data = data, node-next = next))

Example Unfolding

- Example $r = p \rightarrow \text{data}$; requires $p \oplus \text{fo-node-data} \mapsto \dots$

⇒ Heap matching applies rules and leaves

$$\begin{aligned} & \llbracket v = (\downarrow \text{node-data} = \text{data}, \text{node-next} = \text{next}) \downarrow; \\ & \quad (p \oplus \text{fo-node-data} \mapsto \text{cty-int. data} \star \\ & \quad p \oplus \text{fo-node-next} \mapsto \text{cty-ptr. next}) \\ & \quad \text{heap} \\ & \rrbracket \implies \text{data} = \text{node-data } v \end{aligned}$$

- Post-condition requires “re-assembled” heap

$$\begin{aligned} & (p \oplus \text{fo-node-data} \mapsto \text{cty-int. data} \star \\ & \quad p \oplus \text{fo-node-next} \mapsto \text{cty-ptr. next}) \text{ heap} \\ & \implies \text{node } p (\downarrow \text{node-data} = \text{data}, \text{node-next} = \text{next}) \text{ heap} \end{aligned}$$

⇒ Apply unfolding rules “backwards” in heap tactic

- Application of unfolding rules leaves \mathcal{E}, \cdot within heap
- \Rightarrow Apply rewriting rules to “pull out” these symbols
 - ” $\bigwedge P Q. (\mathcal{E} x. P x) \star Q = (\mathcal{E} x. P x \star Q)$ ”
 - ” $\bigwedge P Q. P \star (\mathcal{E} x. Q x) = (\mathcal{E} x. P \star Q x)$ ”
 - ” $\bigwedge P Q. [[\mathcal{E} x. P x]] = (\exists x. [[P x]])$ ”
 - ” $\bigwedge P Q c. (P \cdot c) \star Q = ((P \star Q) \cdot c)$ ”
 - ” $\bigwedge P Q c. P \star (Q \cdot c) = ((P \star Q) \cdot c)$ ”
 - ” $\bigwedge P Q. [[P \cdot c]] = ([[P]] \wedge c)$ ”
- . . . and to make spatial fragment a linear list
 - ” $\bigwedge P Q R. (P \star Q) \star R = (P \star (Q \star R))$ ”
- \Rightarrow Heap is always in very simple form, ready for matching

Summary: Verification by Symbolic Execution

- Use spatial operations in assertions
- Apply `Simpl vcg` to generate verification condition
- Apply `prep_vc` to re-construct execution order
- Apply `step/run` for symbolic execution
- Match resulting heap against post-condition with `heap`
- Prove pure side-conditions on heap values

- Recursive definition: enumerate nodes

```

fun list :: "addr  $\Rightarrow$  int list  $\Rightarrow$  addr  $\Rightarrow$  hasn"
where
  "list p [] q = (emp  $\cdot$  (p = q))"
  | "list p (x # xs') q = ( $\mathcal{E}$  v. node p v  $\star$  list (node-next v) xs' q
     $\cdot$  x = node-data v)"
  
```

- Many programs access first node \Leftrightarrow auto-unfold

```

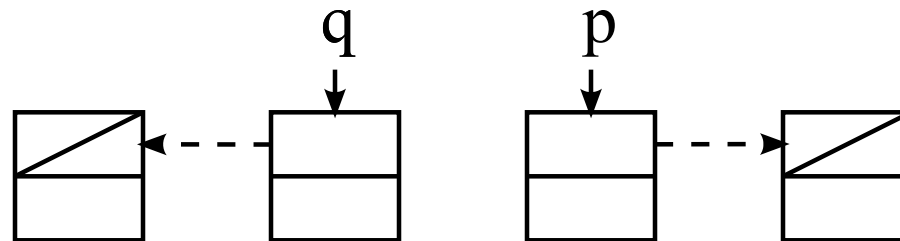
lemma list-step[sep-unfold]:
  "p  $\neq$  q  $\implies$  list p xs q = ( $\mathcal{E}$  v xs'. node p v  $\star$  (list (node-next v) xs' q)
     $\cdot$  (xs = node-data v # xs'))"
  
```


Example: List-Reversal

```

pre: " (list p XS NULL) heap"
post: " (list q (rev XS) NULL) heap"
" q = null;
/*@
  (E xs ys. (list p xs NULL * list q ys NULL) . XS = (rev ys @ xs)) heap
*/
while (p != null) {
  t = p->next;
  p->next = q;
  q = p;
  p = t;
}"

```



```
apply vcg
apply simp-all
apply prep-vc
apply heap
apply run
apply simp
apply heap
apply simp
apply run
apply simp
apply heap
apply simp
done
```

- heap instantiates quantifiers
- ⇒ No need for `apply (rule-tac x="..." in ex1)`

```
q = null;
//@ xs = XS
//@ ys = []
/*@
(list p xs NULL * list q ys NULL • (XS = (rev ys @ xs))) heap
*/
while (p != null) {
  t = p->next;
  p->next = q;
  q = p;
  p = t;
  //@ ys = hd xs # ys
  //@ xs = tl xs
}
```

- Proof: similar
- Advantage: less proof search in heap

- Tool for symbolic execution in separation logic
 - Target: heap-manipulating programs
 - Mostly automatic proofs
 - Disjointness considerations solved syntactically
 - Beyond “records on heap” memory model
- Techniques
 - Symbolic heaps & heap matching
 - Automatic unfolding
 - Solving heap-related proof obligations
 - Reconstruction of execution order from Simpl VC

References

- [1] Andrew W. Appel and Sandrine Blazy. Separation logic for small-step C minor. In Klaus Schneider and Jens Brandt, editors, *Theorem Proving in Higher Order Logics, 20th International Conference*, volume 4732 of *LNCS*. Springer, 2007.
- [2] Josh Berdine, Cristiano Calcagno, and Peter W. O'Hearn. Smallfoot: Modular automatic assertion checking with separation logic. In Frank S. de Boer, Marcello M. Bonsangue, Susanne Graf, and Willem P. de Roever, editors, *Formal Methods for Components and Objects, 4th International Symposium (FMCO)*, volume 4111 of *LNCS*. Springer, 2005.
- [3] Josh Berdine, Cristiano Calcagno, and Peter W. O'Hearn. Symbolic execution with separation logic. In Kwangkeun Yi, editor, *Programming Languages and Systems, Third Asian Symposium (APLAS)*, volume 3780 of *LNCS*. Springer, 2005.
- [4] Matko Botincan, Matthew Parkinson, and Wolfram Schulte. Separation logic verification of c programs with an smt solver. In *4th International Workshop on Systems Software Verification (SSV)*, volume 254. Elsevier, 2009.
- [5] Dino Distefano and Matthew J. Parkinson J. jStar: towards practical verification for Java. *SIGPLAN Not.*, 43(10):213–226, 2008.
- [6] Holger Gast. Semi-automatic proofs about object graphs in separation logic. In Stephan Merz and Gerald Lüttgen, editors, *Automated Verification of Critical Systems (AVoCS '12)*, 2012.
- [7] Christoph Lüth and Dennis Walter. Certifiable specification and verification of C programs. In Ana Cavalcanti and Dennis Dams, editors, *FM 2009: Formal Methods, Second World Congress*, volume 5850 of *LNCS*. Springer, 2009.
- [8] Andrew McCreight. *The Mechanized Verification of Garbage Collector Implementations*. PhD thesis, Department of Computer Science, Yale University, December 2008.
- [9] Andrew McCreight. Practical tactics for separation logic. In S. Berghofer, T. Nipkow, C. Urban, and M. Wenzel, editors, *Theorem Proving in Higher Order Logics, 22nd International Conference (TPHOLs)*, volume 5674 of *LNCS*. Springer, 2009.
- [10] Harvey Tuch, Gerwin Klein, and Michael Norrish. Types, bytes, and separation logic. In Martin Hofmann and Matthias Felleisen, editors, *Proc. 34th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL'07)*, 2007.
- [11] Thomas Tuerk. A formalisation of Smallfoot in HOL. In S. Berghofer, T. Nipkow, C. Urban, and M. Wenzel, editors, *Theorem Proving in Higher Order Logics, 22nd International Conference (TPHOLs)*, volume 5674 of *LNCS*. Springer, 2009.