

### Exercise 1 ( $\lambda$ -Terms)

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the  $\lambda$ -calculus given in the lecture (without any shortcut notations).

c)  $ux(yz)(\lambda v.vy)$

d)  $(\lambda xyz.xz(yz))uvw$

Rewrite the following terms such that there are as few parentheses as possible, and apply all shortcut notation from the lecture:

e)  $((u(\lambda x.(v(wx))))x)$

f)  $((((w(\lambda x.(\lambda y.(\lambda z.((xz)(yz))))))u)v)$

Evaluate the following substitutions:

a)  $(\lambda y.x(\lambda x.x)) [(\lambda y.xy)/x]$

b)  $(y(\lambda v.xv)) [(\lambda y.vy)/x]$

### Exercise 2 ( $\lambda$ -Terms as Trees)

Rewrite the the  $\lambda$ -terms resulting from exercises 1 e) and 1 f) to their corresponding representation as a tree.

### Exercise 3 (Formalizations with $\lambda$ -Terms)

Express the following propositions as  $\lambda$ -terms. Use the constant D as a derivative operator.

a) The derivative of  $x^2$  is  $2x$ .

b) The derivative of  $x^2$  at 3 is 6.

c) Let  $f$  be a function, and let  $g$  be defined as  $g(x) := f(x^2)$ . The derivative of  $g$  at  $x$  is different from the derivative of  $f$  at  $x^2$ .

d) Formulate the proposition c) without using the auxiliary function symbol  $g$ .

#### Homework 4 (Interpreting $\lambda$ -Terms)

Give a *compact* natural-language description of the computational effect of the following  $\lambda$ -terms:

- a)  $\lambda x.x$
- b)  $\lambda xy.x$
- c)  $\lambda xyz.xzy$
- d)  $\lambda xy.x(xy)$
- e)  $\lambda xyz.x(yz)$

#### Homework 5 (Free and Bound Variables)

Mark the free variables in the following examples. Graphically indicate (by drawing arrows) the binding  $\lambda$  for each bound variable.

- a)  $\lambda xyz.(\lambda xy.zx)y(xz)$
- b)  $\lambda x.\lambda y.(\lambda y.z(\lambda z.yx))(\lambda xz.xyz)yx$

#### Homework 6 (Substitutions)

Evaluate the following substitutions:

- a)  $((\lambda x.fx)(\lambda f.fx))[g\ x/f]$
- b)  $(\lambda f.\lambda y.fxy)[f\ y/x]$

#### Homework 7 (Properties of Substitution)

- a) Give a counterexample for

$$s[t/x][u/y] = s[u/y][t/x].$$

- b) Under which conditions does

$$s[t/x][u/y] = s[t[u/y]/x]$$

hold?