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Lambda Calculus
Winter Term 2022/23
Exercise Sheet 1

Exercise 1 (λ-Terms)

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the λ-calculus given in the lecture (without any shortcut notations).

a) $u\, x\, (y\, z)\, (\lambda v.\ v\, y)$

b) $(\lambda x\, y\, z.\ x\, z\, (y\, z))\, u\, v\, w$

Rewrite the following terms such there are as few parentheses as possible, and apply all shortcut notation from the lecture:

c) $((u\, (\lambda x.\ (v\, (w\, x))))\, x)$

d) $(((w\, (\lambda x.\ (\lambda y.\ (\lambda z.\ ((x\, z)\, (y\, z))))))\, u)\, v)$

Evaluate the following substitutions:

e) $(\lambda y.\ x\, (\lambda x.\ x))\, [(\lambda y.\ x\, y)/x]$

f) $(y\, (\lambda v.\ x\, v))\, [(\lambda y.\ v\, y)/x]$

Exercise 2 (λ-Terms as Trees)

Rewrite the λ-terms resulting from exercises 1c) and 1d) to their corresponding representation as a tree.

Exercise 3 (Formalisations with λ-Terms)

Express the following propositions as λ-terms. Use the constant D as a derivative operator.

a) The derivative of x^2 is $2x$.

b) The derivative of x^2 at 3 is 6.

c) Let f be a function, and let g be defined as $g(x) := f(x^2)$. The derivative of g at x is different from the derivative of f at x^2.

d) Formulate the proposition c) without using the auxiliary function symbol g.

Homework 4 (Interpreting λ-Terms)

Give a *compact* natural-language description of the computational effect of the following λ-terms.

a) $\lambda x.\ x$

b) $\lambda x\, y.\ x$

c) $\lambda x\, y\, z.\ x\, z\, y$

d) $\lambda x\, y.\ x\, (x\, y)$

e) $\lambda x\, y\, z.\ x\, (y\, z)$

Homework 5 (Free and Bound Variables)

Mark the free variables in the following examples. Graphically indicate (by drawing arrows) the binding λ for each bound variable.

a) $\lambda x\, y\, z.\ (\lambda x\, y.\ z\, x)\, y\, (x\, z)$

b) $\lambda x.\ \lambda y.\ (\lambda y.\ z\, (\lambda z.\ y\, x))\, (\lambda x\, z.\ x\, y\, z)\, y\, x$

Homework 6 (Substitutions)

Evaluate the following substitutions:

a) $((\lambda x.\ f\, x)\, (\lambda f.\ f\, x))\, [g\, x/f]$

b) $(\lambda f.\ \lambda y.\ f\, x\, y)\, [f\, y/x]$

Homework 7 (Properties of Substitution)

Evaluate the following substitutions:

a) Give a counterexample for

$$s[t/x][u/y] = s[u/y][t/x].$$

b) Under which conditions does

$$s[t/x][u/y] = s[t[u/y]/x]$$

hold?