

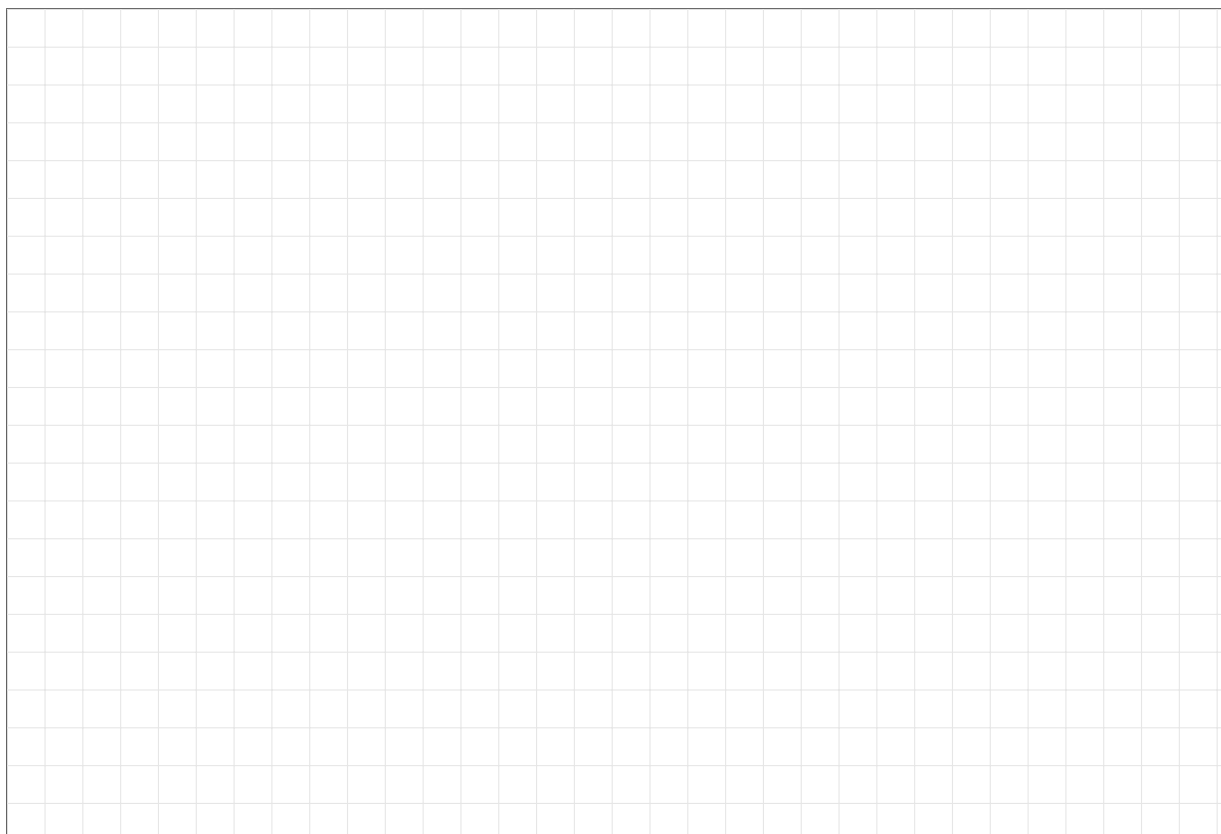
Exercise 1 (Confluence of β -Reduction)

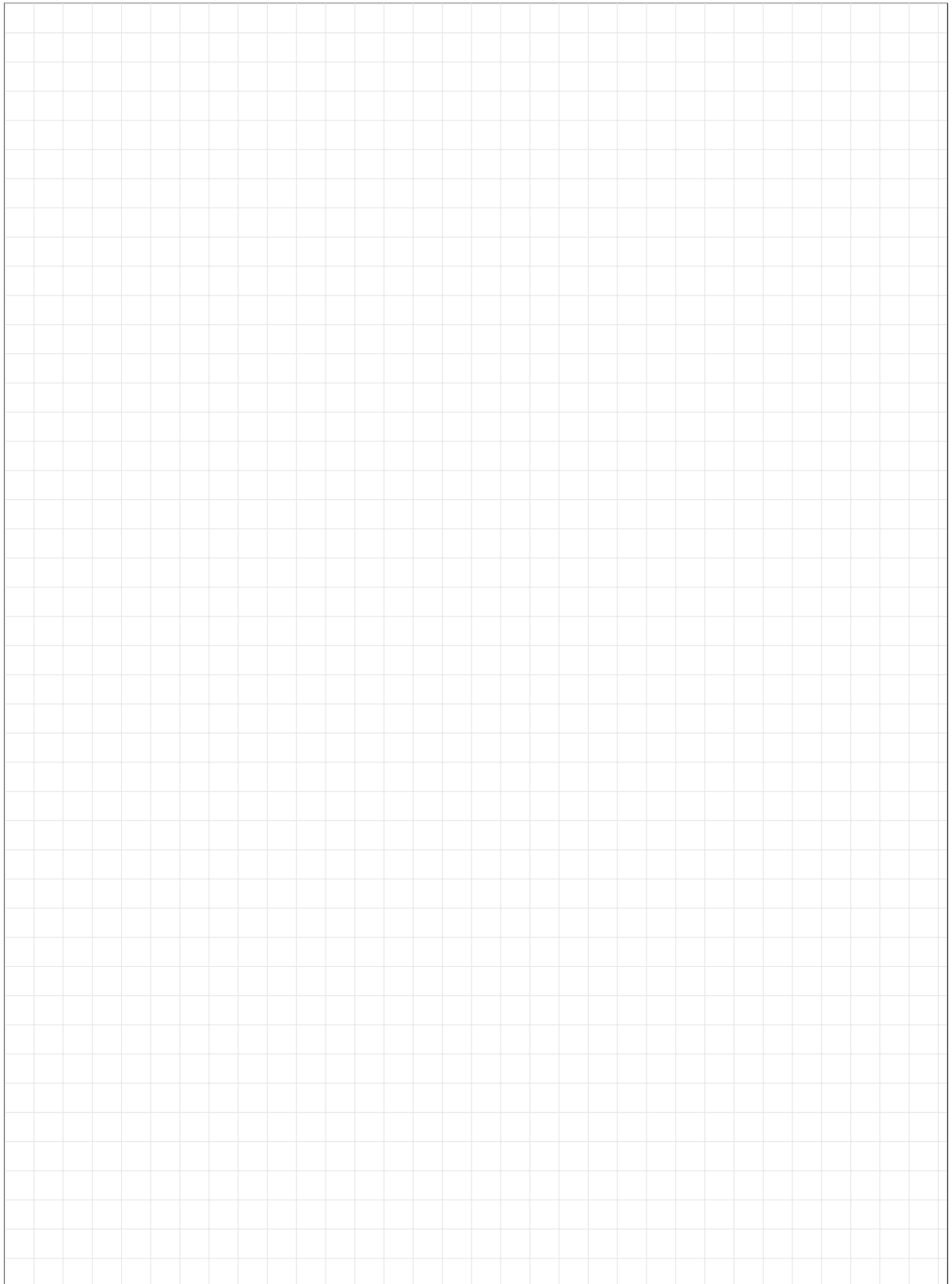
In the lecture we have shown the confluence of \rightarrow_β using the diamond property of parallel β -reduction. In this exercise, we develop an alternative proof.

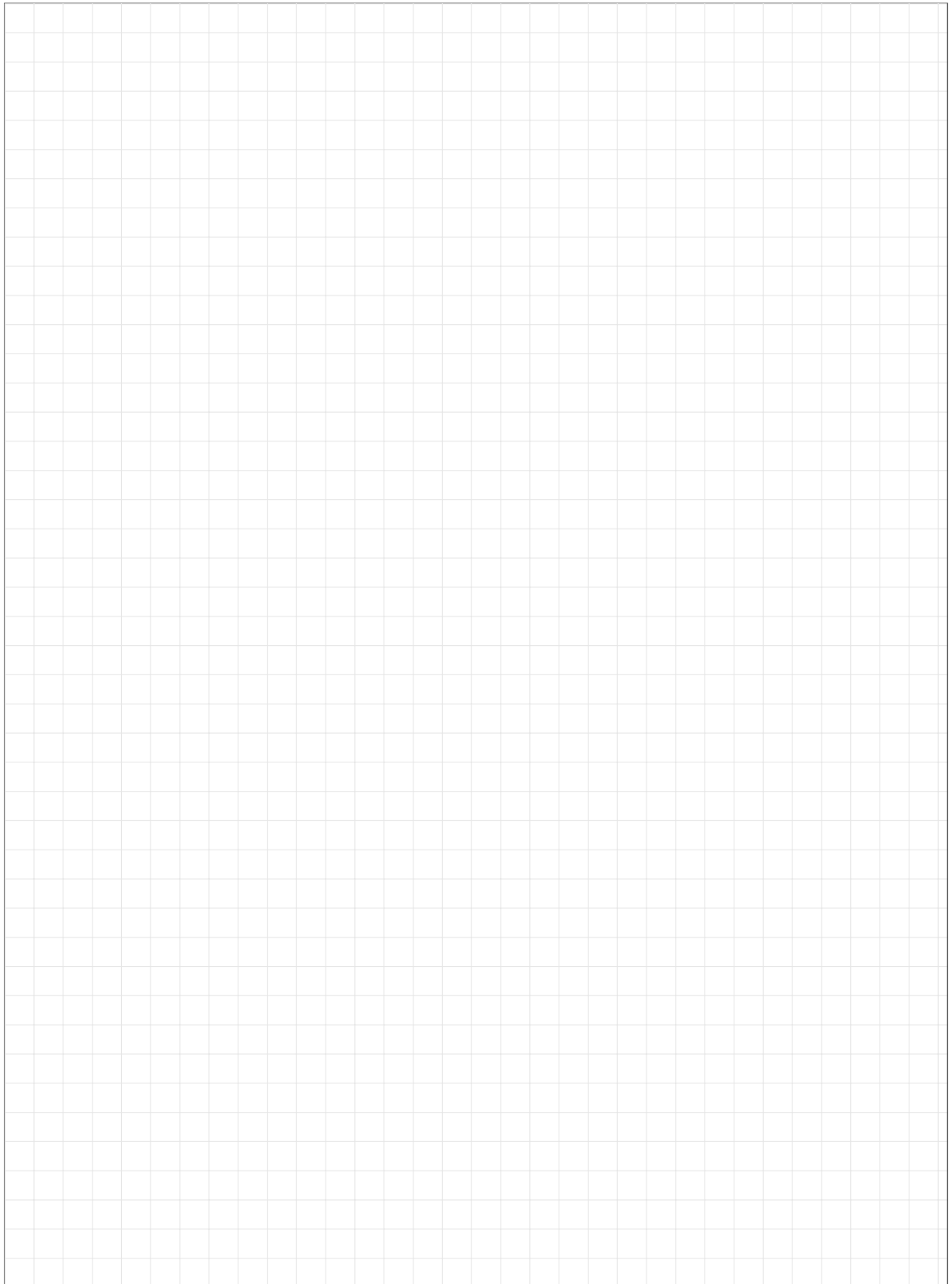
We define the operation $-^*$ on λ -terms inductively over the structure of terms:

$$\begin{aligned}x^* &= x \\(\lambda x. t)^* &= \lambda x. t^* \\(t_1 t_2)^* &= t_1^* t_2^* \quad \text{if } t_1 t_2 \text{ is not a } \beta\text{-redex.} \\((\lambda x. t_1) t_2)^* &= t_1^*[t_2^*/x]\end{aligned}$$

- Show that we have for two arbitrary λ -terms s and t : $s > t \implies t > s^*$
- Show that \rightarrow_β is confluent.



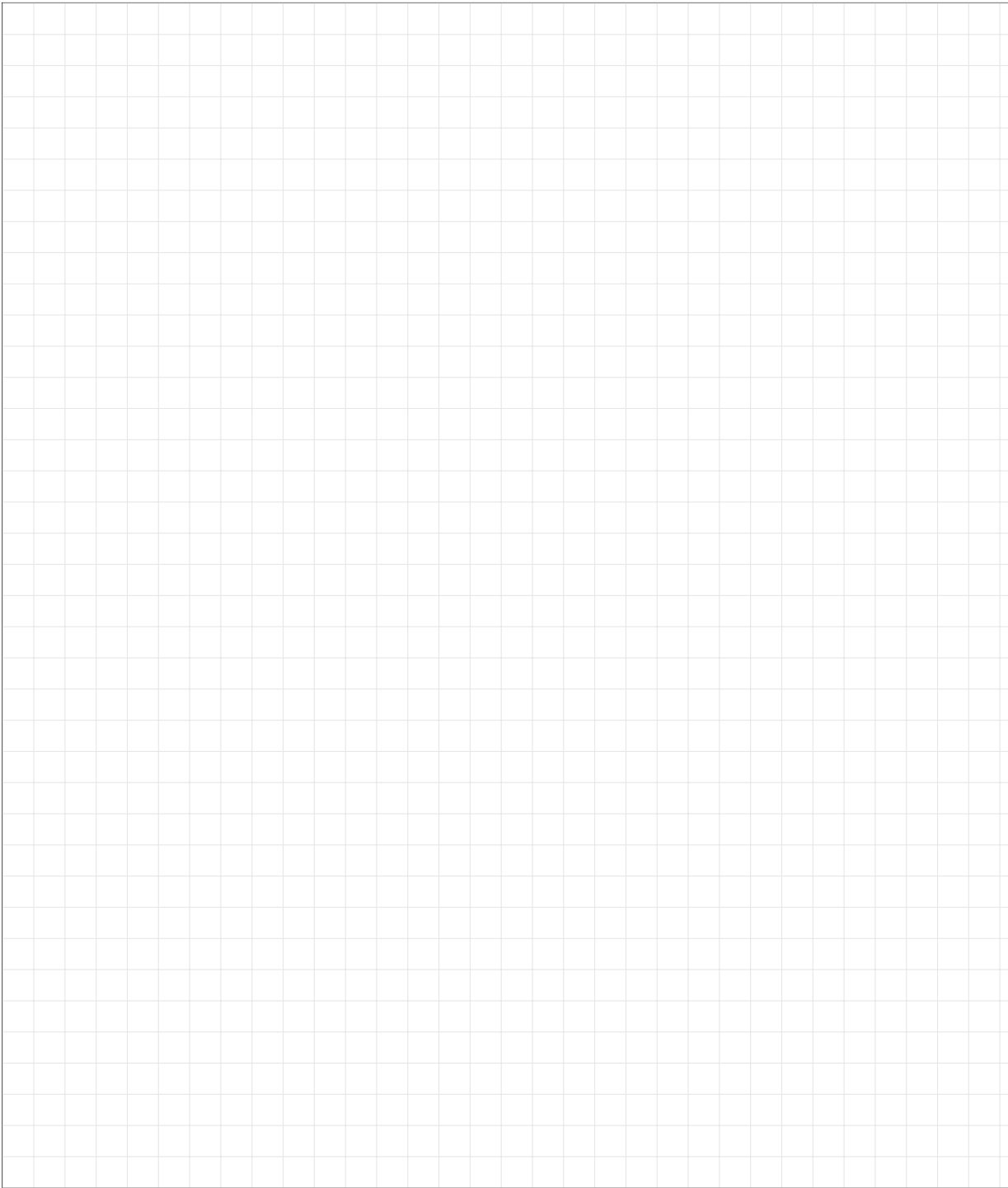




Exercise 2 (Parallel Beta Reduction)

Show:

$$s > t \implies s \rightarrow_{\beta}^* t$$



Exercise 3 (Predecessor and Tail)

- a) Define a predecessor function `pred` on church numerals.
- b) Use the same idea to define `tl` on the fold encoding for lists.



Homework 4 (Parallel Beta Reduction & Substitution)

Show:

$$s > s' \wedge t > t' \implies s[t/x] > s'[t'/x]$$

Homework 5 (Equivalence modulo β -conversion)

Assume that we add the additional axiom

$$\lambda x y. x =_{\beta} \lambda x y. y$$

.

- a) Show that under this assumption $t =_{\beta} t'$ for all t, t' .
- b) Repeat the same for the axiom $\lambda x. x =_{\beta} \lambda x y. y x$.

Homework 6 (Böhm's Theorem)

Böhm's Theorem states that for arbitrary closed terms $M \neq N$ without constant atoms in $\beta\eta$ -normal form, there exist $n \geq 0$ and L_1, \dots, L_n such that:

$$M L_1 \dots L_n x y \rightarrow_{\beta}^* x \text{ and } N L_1 \dots L_n x y \rightarrow_{\beta}^* y.$$

That is, we can tell M and N apart. Show the following two special cases:

- a) $M = \lambda x y z. x z (y z)$ and $N = \lambda x y z. x (y z)$
- b) $M = \lambda x y. x (y y)$ and $N = \lambda x y. x (y x)$