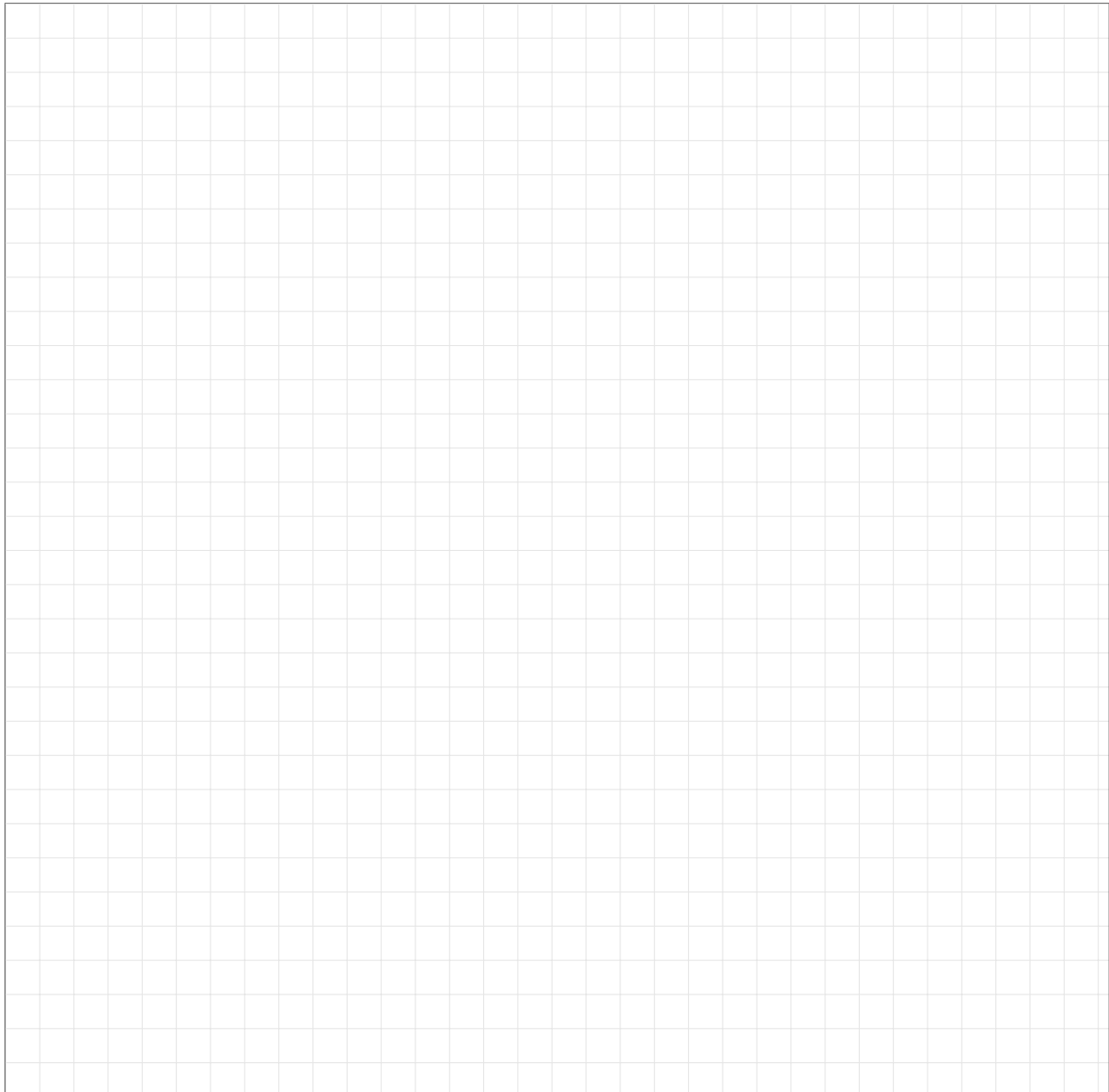


**Exercise 1 (Confluence & Commutation)**

Show: If  $\rightarrow_1$  and  $\rightarrow_2$  are confluent, and if  $\rightarrow_1^*$  and  $\rightarrow_2^*$  commute, then  $\rightarrow_{12} := \rightarrow_1 \cup \rightarrow_2$  is also confluent.

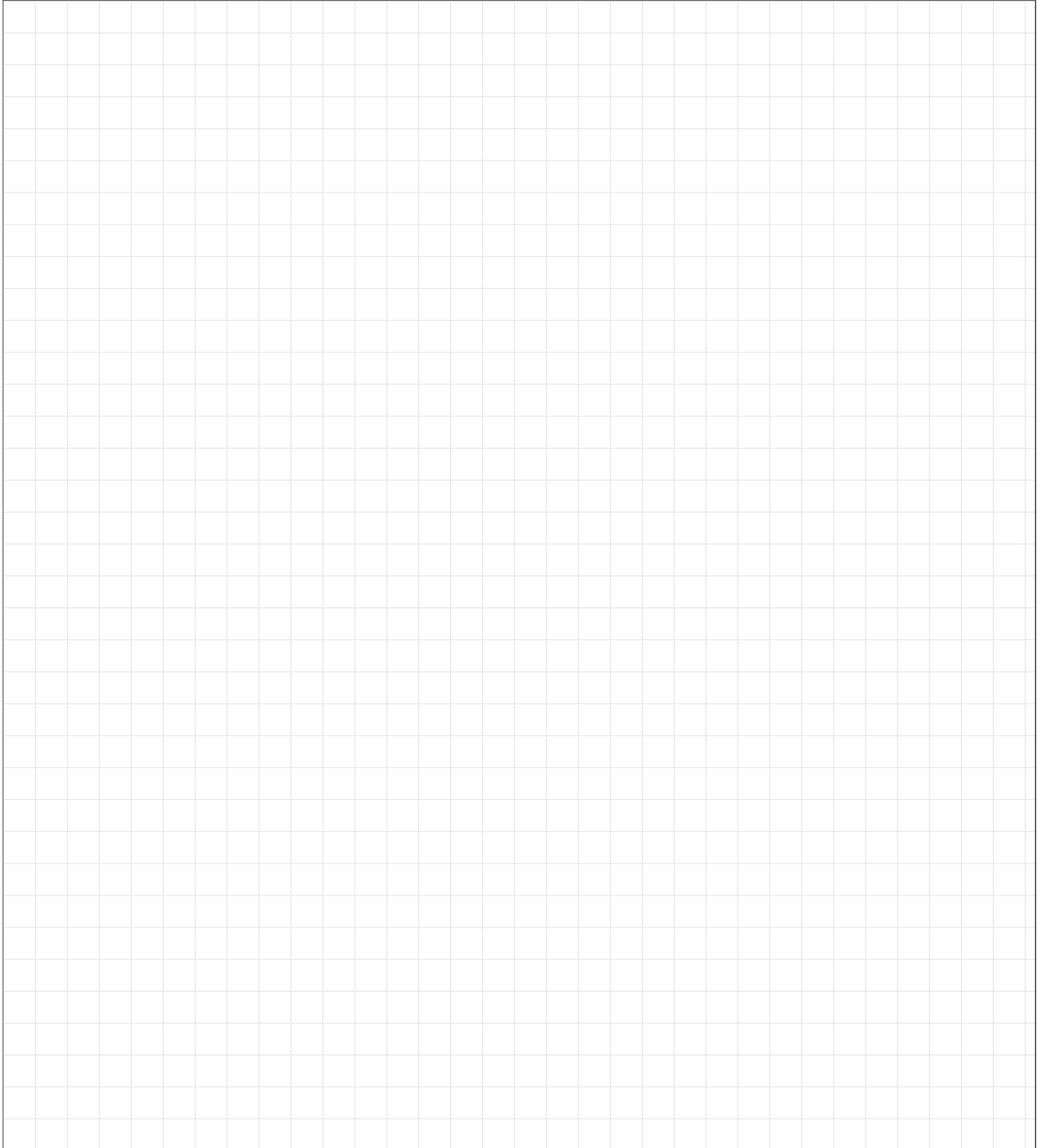


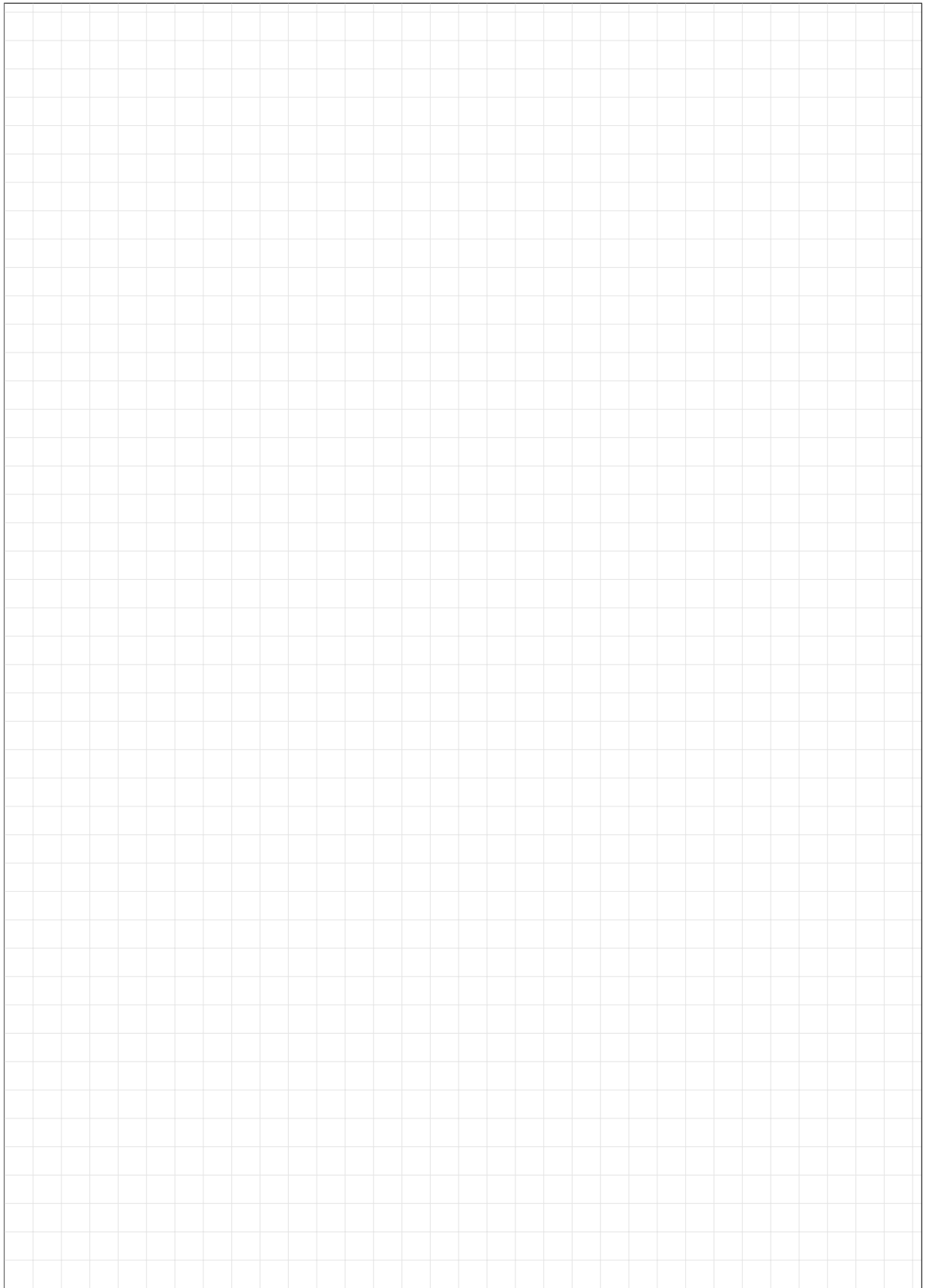
## Exercise 2 (Strong Confluence)

A relation  $\rightarrow$  is said to be *strongly confluent* iff:

$$t_2 \leftarrow s \rightarrow t_1 \implies \exists u. t_2 \rightarrow^* u \leftarrow t_1$$

Show that every *strongly confluent* relation is also *confluent*.



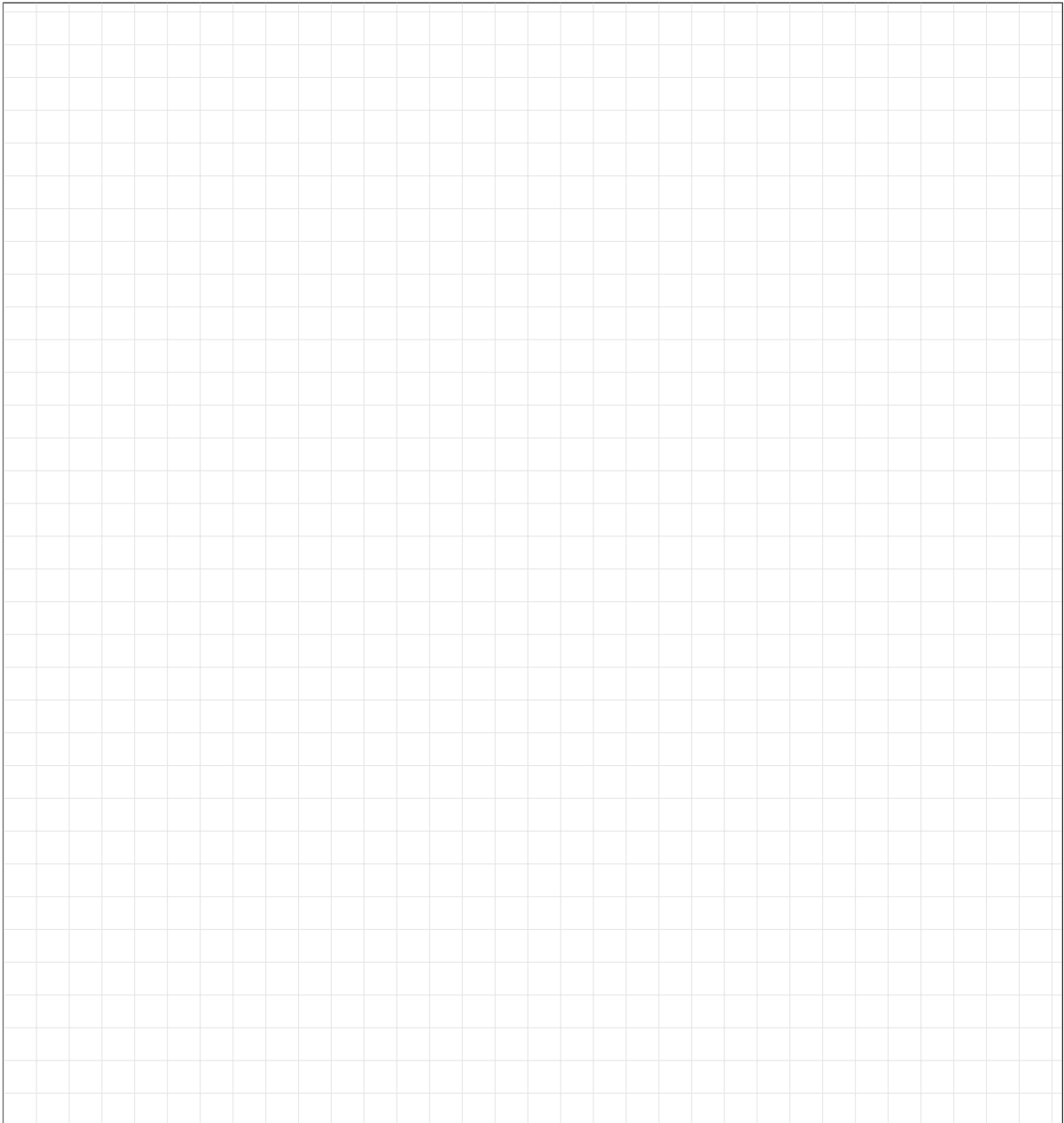


### Exercise 3 (Local Confluence of $\eta$ -reduction)

Analogously to  $\beta$ -reduction, we define  $\eta$ -reduction inductively:

1.  $x \notin \text{FV}(s) \implies (\lambda x. s x) \rightarrow_{\eta} s$
2.  $s \rightarrow_{\eta} s' \implies s t \rightarrow_{\eta} s' t$
3.  $s \rightarrow_{\eta} s' \implies t s \rightarrow_{\eta} t s'$
4.  $s \rightarrow_{\eta} s' \implies (\lambda x. s) \rightarrow_{\eta} (\lambda x. s')$

The proof of local confluence of  $\rightarrow_{\eta}$ , i.e. it holds that there exists a  $u$  with  $t_1 \rightarrow_{\eta}^* u \xleftarrow{*}_{\eta} t_2$  if we have  $t_1 \xleftarrow{\eta} s \rightarrow_{\eta} t_2$ , was very informal. Give a proper proof using this definition.



#### Homework 4 (Semi-Confluence)

A relation  $\rightarrow$  is said to be *semi-confluent* iff:

$$t_2 \xrightarrow{*} s \rightarrow t_1 \implies \exists u. t_2 \rightarrow^* u \xrightarrow{*} t_1$$

Show that  $\rightarrow$  is *semi-confluent* if and only if it is *confluent*.

#### Homework 5 (Diamond Property & Normal Forms)

Show that if  $\rightarrow$  has the diamond property, every element is either in normal form or has no normal form.

#### Homework 6 (Weak Diamond Property)

Assume that  $\rightarrow$  has the following weaker diamond property:

$$t_2 \leftarrow s \rightarrow t_1 \wedge t_1 \neq t_2 \implies \exists u. t_2 \rightarrow u \leftarrow t_1.$$

- a) Is it still the case that every element is either in normal form or has no normal form?
- b) Show that if  $t$  has a normal form, then all its reductions to its normal form have the same length.