

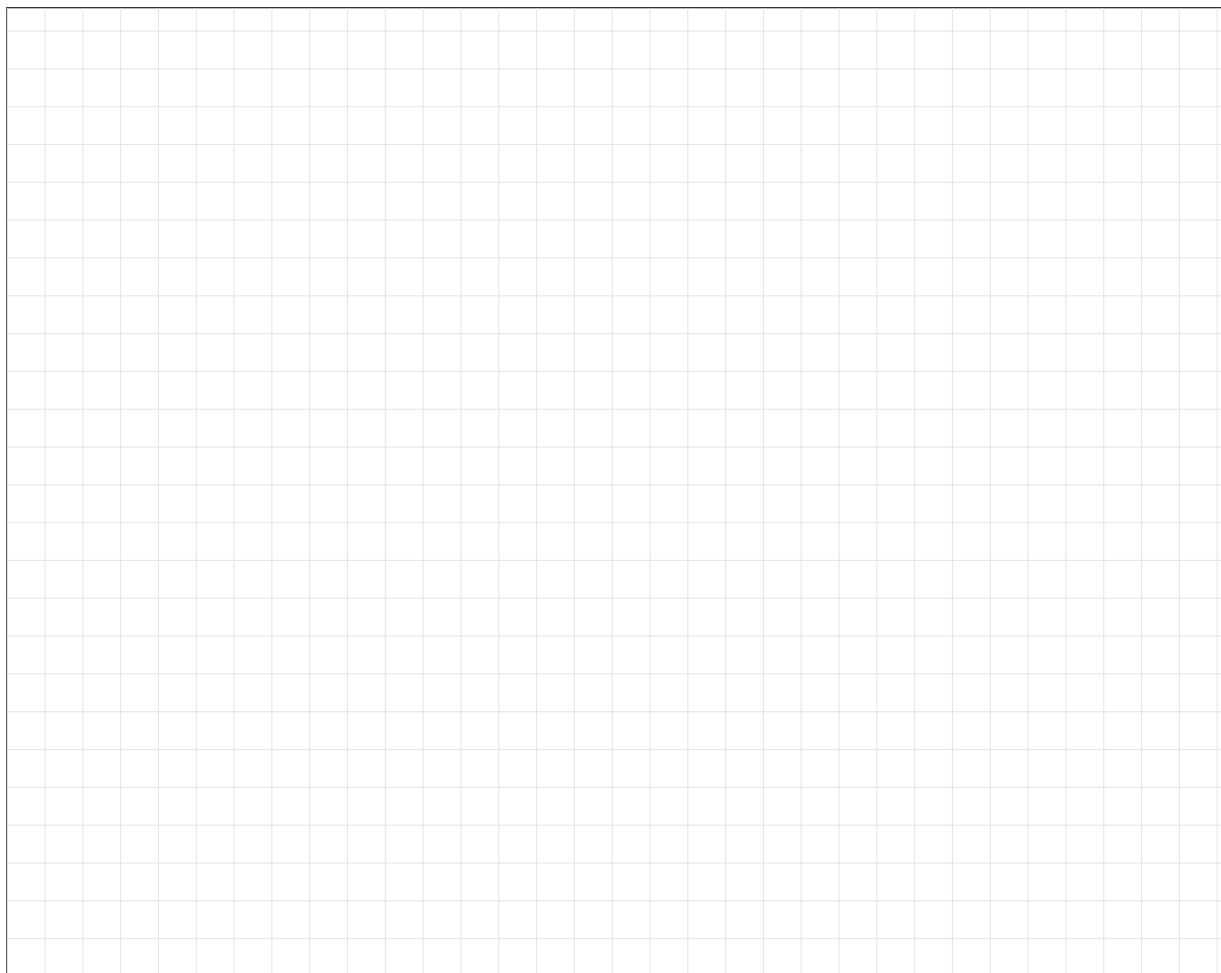
Exercise 1 (Normal Forms)

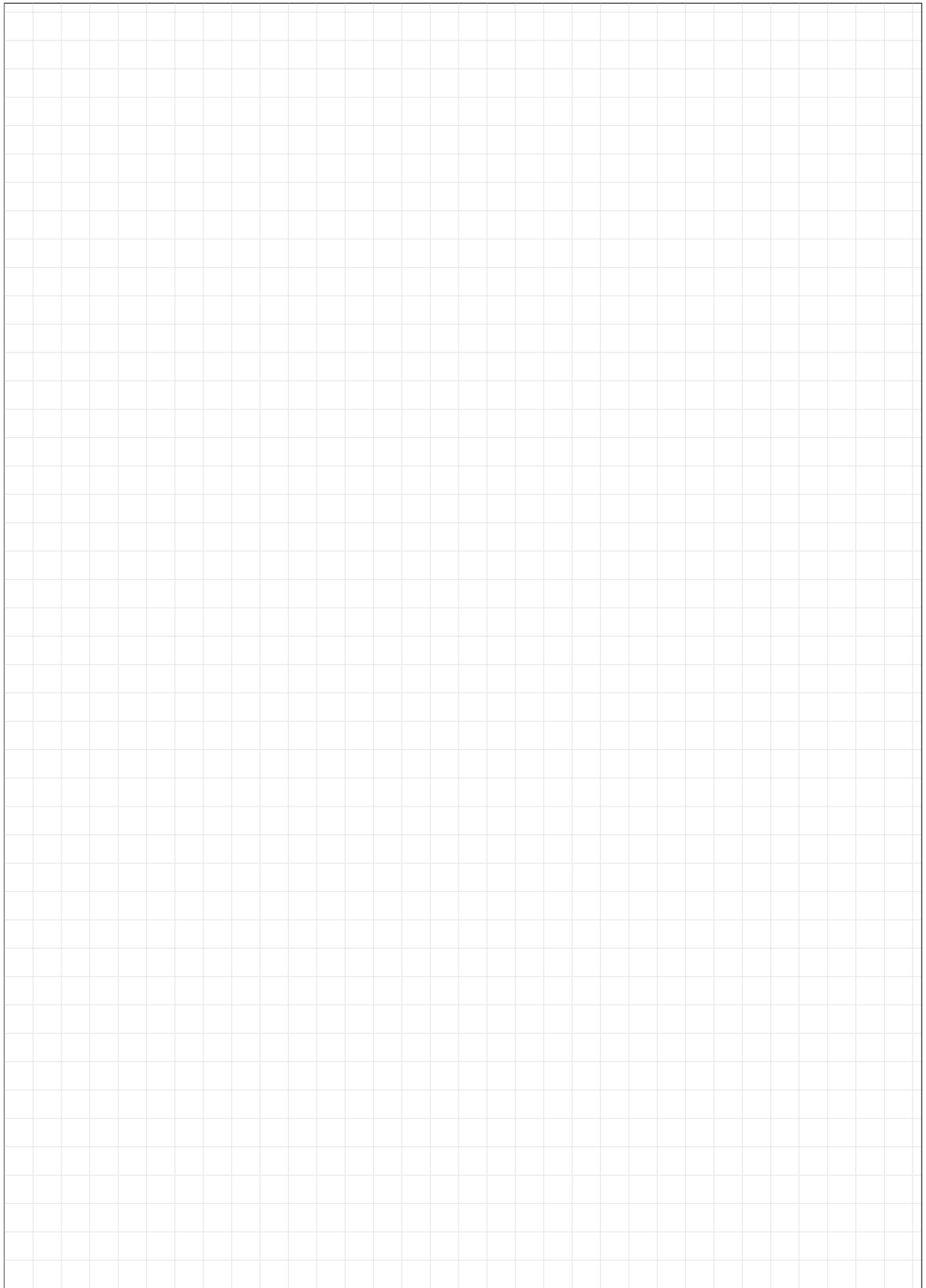
Recall the inductive definition of the set NF of *normal forms*:

$$\frac{\frac{t \in \text{NF}}{\lambda x. t \in \text{NF}}}{\frac{n \geq 0 \quad t_1 \in \text{NF} \quad t_2 \in \text{NF} \quad \dots \quad t_n \in \text{NF}}{x t_1 t_2 \dots t_n \in \text{NF}}}$$

Show that this set precisely captures all normal forms, i.e.:

$$t \in \text{NF} \Leftrightarrow \nexists t'. t \rightarrow_{\beta} t'$$





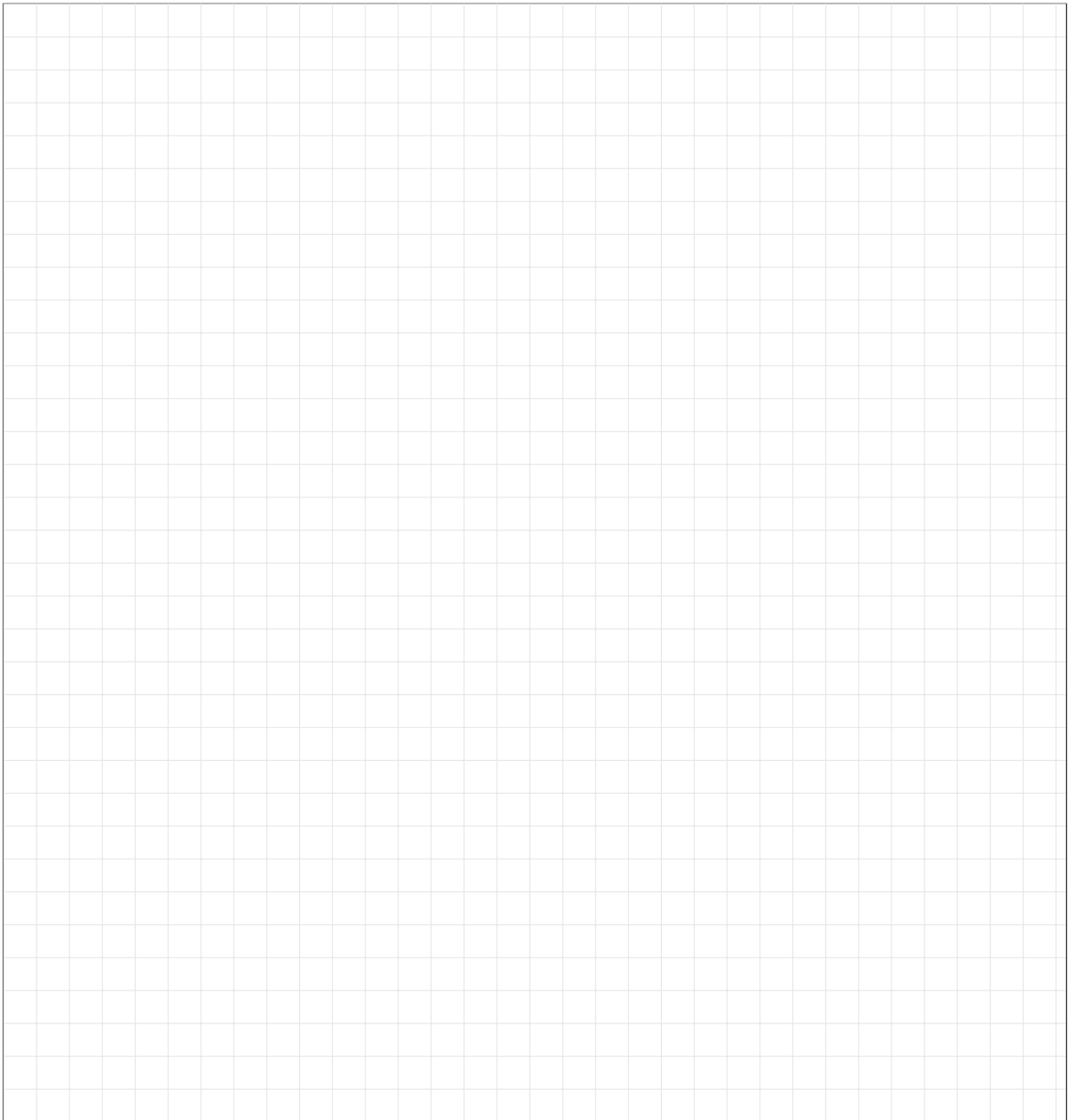
Exercise 2 (Weakly Normalising Terms)

Inductively define the set of weakly normalising terms WN , i.e. the set of terms that have a β -normal form. In particular it should hold that

$$s \in \text{WN} \iff \exists t. s \Rightarrow_n t.$$

Similarly, define the set of strongly normalising terms SN where a term s is strongly normalising if there is no infinite sequence $\{t_i \mid i \in \mathbb{N}\}$ with $s \rightarrow_{\beta}^* t_0$ and $t_i \rightarrow_{\beta} t_{i+1}$ for $i \in \mathbb{N}$.

Give a term t that is weakly but not strongly normalising.



Homework 3 (Normal Forms & Big Step)

Show:

$$t \in \text{NF} \wedge t \Rightarrow_n u \implies u = t$$

Homework 4 (Characterisation of WN)

Prove the following characterisation of WN:

$$t \in \text{WN} \iff \exists t'. t \rightarrow_{\beta}^* t' \wedge t' \in \text{NF}$$

You may use the fact that

$$s \in \text{WN} \iff \exists t. s \Rightarrow_n t$$

and all theorems up to Theorem 1.5.8 from the lecture.