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Lambda Calculus
Winter Term 2023/24
Exercise Sheet 6

Exercise 1 (Normal Forms)

Recall the inductive definition of the set NF of *normal forms*:

$$\frac{t \in \mathrm{NF}}{\lambda x.\ t \in \mathrm{NF}}$$

$$\frac{n \geq 0 \qquad t_1 \in \mathrm{NF} \qquad t_2 \in \mathrm{NF} \qquad \ldots \qquad t_n \in \mathrm{NF}}{x\ t_1\ t_2\ \ldots\ t_n \in \mathrm{NF}}$$

Show that this set precisely captures all normal forms, i.e.:

$$t \in \mathrm{NF} \Leftrightarrow \nexists t'.\ t \to_\beta t'$$

Exercise 2 (Weakly Normalising Terms)

Inductively define the set of weakly normalising terms WN, i.e. the set of terms that have a β-normal form. In particular it should hold that

$$s \in \mathrm{WN} \iff \exists t.\ s \Rightarrow_n t.$$

Similarly, define the set of strongly normalising terms SN where a term s is strongly normalising if there is no infinite sequence $\{t_i \mid i \in \mathbb{N}\}$ with $s \to_\beta^* t_0$ and $t_i \to_\beta t_{i+1}$ for $i \in \mathbb{N}$.

Give a term t that is weakly but not strongly normalising.

Homework 3 (Normal Forms & Big Step)

Show:

$$t \in \mathrm{NF} \wedge t \Rightarrow_n u \Longrightarrow u = t$$

Homework 4 (Characterisation of WN)

Prove the following characterisation of WN:

$$t \in \mathrm{WN} \iff \exists t'.\ t \to_\beta^* t' \wedge t' \in \mathrm{NF}$$

You may use the fact that

$$s \in \mathrm{WN} \iff \exists t.\ s \Rightarrow_n t$$

and all theorems up to Theorem 1.5.8 from the lecture.