Logic Exercises		
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SS 2021	Exercise Sheet 4	07.05.2021

Exercise 4.1. [Natural Deduction]
Prove the following formulas by natural deduction:

1. $(F \wedge G) \wedge H \rightarrow F \wedge (G \wedge H)$
2. $(F \vee G) \vee H \rightarrow F \vee (G \vee H)$
3. $\neg(F \wedge G) \rightarrow (\neg F \vee \neg G)$

Exercise 4.2. [Classical Reasoning]
The intuitionistic version of natural deduction (NI) is obtained from the classical one by replacing the classical rule $(\perp)$ with the rule $\dfrac{\perp}{F}\ (\perp)$.

Show that NI remains complete if we add either of the following rules to it:

- $\dfrac{}{F \vee \neg F}$ (law of excluded middle)
- $\dfrac{\neg\neg F}{F}$ (double negation elimination)

Exercise 4.3. [Curry-Howard]
Bonus: let us explore the Curry-Howard correspondence in an interactive theorem prover.

Homework 4.1. [More Natural Deduction] (++)

Prove the following formulas by natural deduction (as specified in the lecture):

1. $((A \to B) \to A) \to A$
2. $(\neg G \to F) \to (\neg F \to G)$
3. $\neg\neg\neg F \to \neg F$ (while using the $(\bot)$ rule from Exercise 4.2 and not the one defined in the lecture!)

Homework 4.2. [Substitution] (++)

Assume that there are proofs for $\vdash_N G \to G'$ and $\vdash_N G' \to G$. Construct the proof for $\vdash_N F[G/A] \to F[G'/A]$.

Homework 4.3. [Glivenko's Theorem] (+++)

On exercise sheet 3, you already discovered that although some classical laws are not derivable in intuitionistic logic, their doubly negated variants indeed are derivable.

Here's the complete story: Glivenko's Theorem states that F is valid classically if and only if $\neg\neg F$ is valid intuitionistically. Prove Glivenko's Theorem, i.e. show $\vdash_N F \iff \vdash_{NI} \neg\neg F$.

> Taking away the tertium non datur from the mathematician would be about the same as if one would forbid the telescope to the astronomer or the use of his fists to the boxer.
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> — David Hilbert