

# LOGIC EXERCISES

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EXERCISE SHEET 13

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No tutorial on 29.07. Final Q&A and exam preparation tutorial will take place on 01.08, 02.08, or 03.08. Exact place and time will be decided and announced on [Zulip](#).

## Exercise 13.1. [Ferrante–Rackoff Elimination]

Apply the Ferrante–Rackoff Elimination to check the validity of the following sentence:

$$\exists x(\exists y(x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$$

## Exercise 13.2. [Presburger Arithmetic]

Eliminate the quantifiers from the following formulas according to Presburger arithmetic:

1.  $\forall y(3 < x + 2y \vee 2x + y < 3)$
2.  $\forall x(2 \mid x \rightarrow (2x \geq 0 \vee 3x < 2))$

## Exercise 13.3. [Quantifier Elimination for $Th(\mathbb{N}, 0, S, =)$ ]

Give a quantifier-elimination procedure for  $Th(\mathbb{N}, 0, S, =)$  where  $S$  is the successor operation on natural numbers, i.e.  $S(n) = n + 1$ .

*Hint:*  $a = b$  iff  $S^k(a) = S^k(b)$  for any  $a, b, k \in \mathbb{N}$ .

**Homework 13.1.** [Under Presburger] (++)

Perform Presburger arithmetic quantifier elimination for the following formula:

$$\forall x(\exists y(x = 2y \wedge 2 \mid y) \rightarrow 4 \mid x)$$

You may additionally simplify ground atoms during the process.

**Homework 13.2.** [Quantifier Elimination for  $Th(\mathbb{Z}, 0, S, P, =, <)$ ] (+++)

Give a quantifier-elimination procedure for  $Th(\mathbb{Z}, 0, S, P, =, <)$  where  $S$  is the successor and  $P$  the predecessor operation on integers, i.e.  $S(n) = n + 1$  and  $P(n) = n - 1$ . Do not use Presburger arithmetic; give a direct algorithm.

It is always easy to be logical.

It is almost impossible to be logical to the bitter end.

— Albert Camus