Technische Universität München | **Equational Logic**
Institut für Informatik | **Summer Term 2012**
Prof. Tobias Nipkow, Ph.D. | Exercise Sheet 6
Brian Huffman and Peter Lammich | May 25

Homework is due on June 1st, before the tutorial.

Exercise 1 (H) (*Substitutions*)

Let σ and σ' be substitutions. Prove or refute:

a) $\sigma \subseteq \sigma'$ implies $\sigma \lesssim \sigma'$.

b) $\sigma'\sigma' = \sigma'$ and $\sigma \subseteq \sigma'$ implies $\sigma \lesssim \sigma'$.

Note that in $\sigma \subseteq \sigma'$, a substitution is interpreted as a relation:

$$\sigma \subseteq \sigma' :\Longleftrightarrow \mathcal{D}om(\sigma) \subseteq \mathcal{D}om(\sigma') \wedge \sigma = \sigma'|_{\mathcal{D}om(\sigma)}$$

Exercise 2 (H) (*Unification, Matching*)

Do the following unification and matching problems have a solution? Specify the most general unifier or the matcher!

a) $f(x,y) =^? / \lesssim^? f(h(a),x)$

b) $f(x,y) =^? / \lesssim^? f(h(x),x)$

c) $f(x,b) =^? / \lesssim^? f(h(y),z)$

d) $f(x,x) =^? / \lesssim^? f(h(y),y)$

Note that the (syntactic) matching problem $s \lesssim^? t$ is the problem of finding a substitution σ such that $\sigma(s) = t$. If such a σ exists, it is called *matcher*. Matching can easily be reduced to unification, by regarding the variables in t as constants.

Exercise 3 (T) (*Most General Unifier*)

a) Let S and T be unification problems. Moreover, let σ be a most general unifier for S and θ be a most general unifier for $\sigma(T)$. Show that $\theta\sigma$ is a most general unifier for $S \cup T$.

b) Show the following compactness theorem:

> Every satisfiable set of equations over a finite set of variables contains a finite subset that has the same solutions.

Note that equations are interpreted in the term algebra here.

Exercise 4 (T) (*Matching-Algorithm*)

The matching problem can be reduced to the unification problem (see Exercise 2).

In this exercise, you shall modify the transformation rules of the unification algorithm such that they solve the matching problem directly. Note that variables are allowed on both sides of the equations.