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Equational Logic
Summer Term 2012
Exercise Sheet 7
June 8

Homework is due on June 15th, before the tutorial.

Exercise 1 (H) (*Termination*)

Let $\Sigma = \{a, b, c, d\}$, with unary function symbols a, b und c and a constant symbol d. Show that the term rewriting system with the following rules terminates:

$$\begin{aligned} b(a(x)) &\longrightarrow a(b^2(c(x))) \\ c(a(x)) &\longrightarrow a(b(c^2(x))) \\ c(b(x)) &\longrightarrow b(c(x)) \end{aligned}$$

Hint: Consider how the number of occurences of as changes in each step. Then regard the sequences of function symbols in between the as as strings.

Exercise 2 (H) (*Reduction Ordering*)

Recall that a reduction ordering is a well-founded ordering on terms that is compatible with context and closed under substitutions. Now consider the subterm ordering $>_{ST}$, defined so that $s >_{ST} t$ iff t is a proper subterm of s.

a) Show that $>_{ST}$ is no reduction ordering.

b) Show that a term-rewriting system R with $R \subseteq >_{ST}$ always terminates. Here, $R \subseteq >_{ST}$ means that $l >_{ST} r$ for every rewrite rule $(l \longrightarrow r) \in R$.

Exercise 3 (T) (*Termination*)

A term rewriting system R is called *right reduced*, if for all $(l \longrightarrow r) \in R$, the right hand side r is irreducible. Show that every right reduced and right ground term rewriting system terminates.

Hint: Consider the positions in the term at which rules from R may be applied, and specify a suitable order on terms. Is there a simpler way to get this lemma as a corollary from a lemma that was presented in the lecture?

Exercise 4 (T) (*Deciding Termination for Right-Ground TRSs*)

In the lecture, we discussed a procedure to decide termination of right-ground term rewriting systems. It is important that we use a breadth-first search strategy, as you shall demonstrate in this exercise.

Given the following procedure that uses a depth-first approach:

Input A right ground term rewriting system $R = \{l_1 \longrightarrow r_1, \ldots, l_n \longrightarrow r_n\}$.

Procedure Enumerate all reduction sequences that start with r_1, in depth-first order. If one of those sequences contains r_1 as a subterm, output *non-terminating*. Otherwise continue with the sequences starting at r_2, and so on. If all right hand sides have been processed, output *terminating*.

Find a right-ground term rewriting system such that the above procedure does not terminate.