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Equational Logic
Summer Term 2012
Exercise Sheet 10
June 29

Homework is due on July 6th, before the tutorial.

Exercise 1 (H) (λ-*Calculus Interpreter*)

Implement an interpreter for the lambda calculus in your favorite programming language. It shall convert a lambda-term into its β-normal form. If there is an infinite sequence of β-reductions, your program is allowed not to terminate. Comment your program thoroughly. Test your program for (at least) the following examples, and document the results:

$t_1 = (\lambda x.\ (\lambda y.\ x\ y))\ y$ $\quad$ $t_2 = (\lambda x.\ \lambda x.\ x)\ y$ $\quad$ $t_3 = (\mathsf{k}\ c)\ \mathsf{omega}$

$t_4 = (\lambda x.\ ((f\ x)\ x))\ 5$ $\quad$ $t_5 = (\lambda x.\ x)\ (\lambda x.\ x)$ $\quad$ $t_6 = x\ (\lambda y.\ y)$

$t_7 = \mathsf{self}\ (\lambda x.\ (fx))$ $\quad$ $t_8 = ((\mathsf{s}\ \mathsf{k})\ x)\ y$ $\quad$ $t_9 = ((\mathsf{s}\ \mathsf{k})\ \mathsf{k})\ x$

$t_{10} = (\lambda x.\ (\lambda y.\ (x\ \mathsf{i})))\ (\lambda y.\ (x\ y))$ $\quad$ $t_{11} = (\lambda x.\ ((\lambda y.\ (x\ y))\ c))\ (\mathsf{i}\ d)$

where

$\mathsf{self} = (\lambda x.\ (x\ x))$ $\quad$ $\mathsf{omega} = \mathsf{self}\ \mathsf{self}$ $\quad$ $\mathsf{i} = \lambda x.\ x$

$\mathsf{k} = \lambda x.\ (\lambda y.\ x)$ $\quad$ $\mathsf{s} = \lambda x.\ (\lambda y.\ (\lambda z.\ ((x\ z)\ (y\ z))))$

Hints:

- This exercise works best in a functional programming language.
- You are not required to write a parser/pretty printer, but may encode the example terms directly in your program.
- t_1 and t_2 test wether you have implemented substitutions right, there normal forms are $\lambda y'.\ y\ y'$ and $\lambda x.\ x$.

Exercise 2 (H) (*Substitution Lemma*)

Show that:

$$s[t/x][u/y] = s[u/y][t[u/y]/x] \text{ if } x \notin \mathsf{FV}(u)$$

Exercise 3 (T) (λ-*Terms*)

Evaluate the following substitutions:

a) $(\lambda y.x(\lambda x.x))\ [(\lambda y.xy)/x]$ $\quad$ b) $(y(\lambda v.xv))\ [(\lambda y.vy)/x]$

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the λ-calculus given in the lecture (without any shortcut notations).

c) $ux(yz)(\lambda v.vy)$

d) $(\lambda xyz.xz(yz))uvw$

Rewrite the following terms such that there are as few parentheses as possible, and apply all shortcut notation from the lecture:

e) $((u(\lambda x.\,(v(wx))))x)$

f) $(((w(\lambda x.(\lambda y.(\lambda z.((xz)(yz))))))u)v)$

Exercise 4 (T) (*Formalization with λ-Terms*)

Express the following propositions as λ-terms. Use the constant D as a derivative operator.

a) The derivative of x^2 is $2x$.

b) The derivative of x^2 at 3 is 6.

c) Let f be a function, and let g be defined as $g(x) := f(x^2)$. The derivative of g at x is different from the derivative of f at x^2.

d) Formulate the proposition c) without using the auxiliary function symbol g.