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Equational Logic
Summer Term 2015
Exercise Sheet 7

Exercise 34 (Termination)

Let $\Sigma = \{a, b, c, d\}$, with unary function symbols a, b und c and a constant symbol d. Show that the term rewriting system with the following rules terminates:

$$\begin{aligned} b(a(x)) &\longrightarrow a(b^2(c(x))) \\ c(a(x)) &\longrightarrow a(b(c^2(x))) \\ c(b(x)) &\longrightarrow b(c(x)) \end{aligned}$$

Hint: Consider how the number of occurences of as changes in each step. Then regard the sequences of function symbols in between the as as strings.

Exercise 35 (Hilbert's 10th Problem (Exercise 5.8 of TRaAT))

Show that undecidability of Hilbert's 10th Problem implies that the following problem (TRaT Exercise 5.8) is undecidable:

Instance: Two polynomials $P, Q \in \mathbb{N}[X_1, \ldots, X_n]$ in n indeterminates with non-negative integer coefficients, and a (decidable) subset A of $\mathbb{N}$.

Question: Does $P >_A Q$ hold, i.e. is the value of P greater than the value of Q for all valuations with elements in A.

Show that this implies that there exists a polynomial interpretation $\mathcal{A}$ for which it is in general undecidable whether two terms l, r satisfy $l >_{\mathcal{A}} r$.

Homework 36 (Reduction Ordering)

Recall that a reduction ordering is a well-founded ordering on terms that is compatible with context and closed under substitutions. Now consider the subterm ordering $>_{ST}$, defined so that $s >_{ST} t$ iff t is a proper subterm of s.

a) Show that $>_{ST}$ is no reduction ordering.

b) Show that a term-rewriting system R with $R \subseteq >_{ST}$ always terminates. Here, $R \subseteq >_{ST}$ means that $l >_{ST} r$ for every rewrite rule $(l \longrightarrow r) \in R$.

Homework 37 (Polynomial Interpretation)

Use the polynomial interpretation $\mathcal{A}$ with $A = \mathbb{N} - \{0, 1, 2\}$ and $P_f(X, Y) = X^2 + XY$to show that the following term rewriting system terminates:

$$\{\ f(f(x, y), z) \longrightarrow f(x, f(y, z)),\ f(x, f(y, z)) \longrightarrow f(y, y)\ \}$$

Homework 38 (Interpretation)

Prove termination of the following term rewriting system using the interpretation method:

$$\{f(f(x)) \longrightarrow f(g(f(x))\}$$