

### Exercise 51 (Confluence of $\beta$ -Reduction)

In the lecture we have shown the confluence of  $\longrightarrow_\beta$  using the diamond property of parallel  $\beta$ -reduction. In this exercise, we develop an alternative proof.

We define the operation  $*$  on  $\lambda$ -terms inductively over the structure of terms:

$$\begin{aligned} x^* &= x \\ (\lambda x. t)^* &= \lambda x. t^* \\ (t_1 t_2)^* &= t_1^* t_2^* \quad \text{if } t_1 t_2 \text{ is not } \beta\text{-reducible.} \\ ((\lambda x. t_1) t_2)^* &= t_1^* [t_2^*/x] \end{aligned}$$

- Show that we have for two arbitrary  $\lambda$ -terms  $s$  and  $t$ :  $s > t \implies t > s^*$
- Show that  $\longrightarrow_\beta$  is confluent.

### Exercise 52 ( $\alpha$ -Equivalence)

Prove transitivity for  $\alpha$ -equivalence, that is, for all terms  $t_1, t_2, t_3$ ,

$$t_1 =_\alpha t_2 \implies t_2 =_\alpha t_3 \implies t_1 =_\alpha t_3$$

### Homework 53 (Parallel $\beta$ -Reduction)

In the lecture, we defined parallel  $\beta$ -reduction  $>$  inductively as follows:

$$\begin{aligned} s &> s \\ s > s' &\implies \lambda x. s > \lambda x. s' \\ s > s' \wedge t > t' &\implies (s t) > (s' t') \\ s > s' \wedge t > t' &\implies (\lambda x. s) t > s' [t'/x] \end{aligned}$$

We also showed  $> \subseteq \longrightarrow_\beta^*$ . In this exercise, you have to show:  $\longrightarrow_\beta \subseteq >$

*Hint:* Use induction over the inductive definition of  $\longrightarrow_\beta$ .

### Homework 54 (Expanding lets)

We have a language with **let**-expressions, i.e.:

$$t = v \mid t t \mid \mathbf{let} \ v = t \ \mathbf{in} \ t$$

Write a program which expands all **let**-expressions, the **let**-semantics are:

$$(\mathbf{let} \ v = t_1 \ \mathbf{in} \ t_2) = (\lambda v. t_2) t_1$$