# Logics Exercise

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**Submission of Homework:** Before tutorial on May 4

**Homework 3.1.** **[Equivalence]** (4 points)
Let $F$ and $G$ be arbitrary formulas. (In particular, they may contain free occurrences of $x$.) Which of the following equivalences hold? Proof or counterexample!

1. $\forall x(F \wedge G) \equiv \forall x F \wedge \forall x G$
2. $\exists x(F \wedge G) \equiv \exists x F \wedge \exists x G$

**Homework 3.2.** **[Preorders]** (4 points)
A reflexive and transitive relation is called *preorder*. In predicate logic, preorders can be characterized by the formula

$$F \equiv \forall x \forall y \forall z\ (P(x,x) \wedge (P(x,y) \wedge P(y,z) \longrightarrow P(x,z)))$$

Which of the following structures are models of $F$? No proofs are required for the positive case. Give counterexamples for the negative case!

1. $U^{\mathcal{A}} = \mathbb{N}$ and $P^{\mathcal{A}} = \{(m,n) \mid m = n\}$
2. $U^{\mathcal{A}} = 2^{\mathbb{N}}$ and $P^{\mathcal{A}} = \{(A,B) \mid A \supseteq B\}$
3. $U^{\mathcal{A}} = \mathbb{Z}$ and $P^{\mathcal{A}} = \{(x,y) \mid\ 5 > |x-y|\ \}$

**Homework 3.3.** **[Infinite Models]** (5 points)
Consider predicate logic with equality. We use infix notation for equality, and abbreviate $\neg(s = t)$ by $s \neq t$. Moreover, we call a structure finite iff it's universe is finite.

1. Specify a finite model for the formula $\forall x\ (c \neq f(x) \wedge x \neq f(x))$.
2. Specify a model for the formula $\forall x \forall y\ (c \neq f(x) \wedge (f(x) = f(y) \longrightarrow x = y))$.
3. Show that the above formula has no finite model.

**Homework 3.4.** **[Normal Forms]** (3 points)
Convert the following formula to Skolem form:

$$P(x) \wedge \forall x\ (Q(x) \wedge \forall x \exists y\ P(f(x,y))$$

Show at least the main intermediate conversion stages.

**Homework 3.5.** **[Relation to Propositional Logic]** (4 points)
Suppose that formula $F$ does not contain any variables or quantifiers. Your task is to construct a *propositional* formula $G$ such that $F$ is valid iff $G$ is valid. Proof that your construction does indeed fulfill this property. Is it also the case that $F$ is satisfiable iff $G$ is satisfiable?

*Hints*: The approach should define a new *atom* for every *atomic formula* in $F$. To construct a structure for $F$ from an assignment for $G$, it may be helpful to use as your universe the set of all terms which can be constructed from function symbols in $F$. You can assume that $F$ contains at least one constant to ensure that this universe is non-empty.