

# LOGICS EXERCISE

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EXERCISE SHEET 13

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**Submission of Homework:** Before tutorial on July 13

**Exercise 13.1. [Relations]**

Use natural deduction to show: Let  $T$  be a total relation, and  $A$  be an antisymmetric relation, and  $T \subseteq A$ , then  $A \subseteq T$ .

1. Express the above statement in first-order logic
2. Prove it by natural deduction

**Exercise 13.2. [Proof!?!]**

Consider the following “proof” of an invalid sequent:

$$\frac{\frac{\exists y P(y, y), \forall x \exists y P(x, y) \Rightarrow \exists y P(y, y)}{\forall x \exists y P(x, y) \Rightarrow \exists y P(y, y)} \text{axiom}}{\forall L \text{ with } [y/x]}$$

What went wrong?

**Exercise 13.3. [Translation from Sequent Calculus to Natural Deduction]**

Show that  $\vdash_G \Gamma \Rightarrow \Delta$  implies  $\Gamma \vdash_N \bigvee \Delta$ . Only consider the cases for axioms and  $\wedge$ .

**Exercise 13.4. [Natural Deduction with Equality]**

Prove by natural deduction:

1.  $f^3(a) = a \wedge f^5(a) = a \rightarrow f(a) = a$
2.  $A[t/x] \rightarrow \forall x(x = t \rightarrow A)$  if  $x$  does not occur free in  $t$

**Homework 13.1.** [Hintikka Sets in Predicate Logic] (6 points)

A Hintikka Set (of predicate logic) for a signature  $\Sigma$  is a set  $H$  of formulae, such that:

1. For any atom  $A$ , not both  $A \in H$  and  $\neg A \in H$
2. If  $\neg\neg Z \in H$  then also  $Z \in H$
3. If  $F_1 \wedge F_2 \in H$  then also  $F_1 \in H$  and  $F_2 \in H$
4. If  $\neg(F_1 \vee F_2) \in H$  then also  $\neg F_1 \in H$  and  $\neg F_2 \in H$
5. If  $F_1 \vee F_2 \in H$  then also  $F_1 \in H$  or  $F_2 \in H$
6. If  $\neg(F_1 \wedge F_2) \in H$  then also  $\neg F_1 \in H$  or  $\neg F_2 \in H$
7. If  $(\forall x F) \in S$ , then  $F[t/x] \in S$  for all  $\Sigma$ -terms  $t$
8. If  $(\exists x F) \in S$ , then  $F[t/x] \in S$  for some  $\Sigma$ -term  $t$
9. If  $\neg(\forall x F) \in S$ , then  $(\exists x \neg F) \in S$
10. If  $\neg(\exists x F) \in S$ , then  $(\forall x \neg F) \in S$

Show: Every Hintikka-set of predicate logic is satisfiable.

**Homework 13.2.** [Proofs with Sequent Calculus for Predicate Logic] (5 points)

Prove the following formulas in the sequent calculus or construct a counterexample from a stuck proof attempt in the case a formula does not hold:

- $\exists x(P \rightarrow Q(x)) \rightarrow (P \rightarrow \exists z(Q(z)))$
- $\exists x(P \rightarrow Q(x)) \rightarrow (P \rightarrow \forall z(Q(z)))$

**Homework 13.3.** [Natural Deduction and Resolution in Predicate Logic] (5 points)

Show that the following formula is valid – once with natural deduction and once with resolution:

$$\neg(\forall x(\exists y(\neg P(x) \wedge P(y))))$$

**Homework 13.4.** [Left-Sided Sequent Calculus for Predicate Logic] (4 points)

Extend the left-sided sequent calculus for propositional logic from exercise 11.4 with the necessary rules for predicate logic and repeat the correctness proof for the new rules.