

# First-Order Logic

# Compactness

[Harrison, Section 3.16]

## More Herbrand Theory

Recall Gödel-Herbrand-Skolem:

### Theorem

*Let  $F$  be a closed formula in Skolem form. Then  $F$  is satisfiable iff its Herbrand expansion  $E(F)$  is (propositionally) satisfiable.*

Can easily be generalized:

### Theorem

*Let  $S$  be a set of closed formulas in Skolem form. Then  $S$  is satisfiable iff  $E(S)$  is (propositionally) satisfiable.*

## Transforming sets of formulas

Recall the transformation of single formulas into equisatisfiable Skolem form: close, RPF, skolemize

### Theorem

*Let  $S$  be a countable set of closed formulas. Then we can transform it into an equisatisfiable set of closed formulas  $T$  in Skolem form.*

*We call this transformation function skolem.*

- ▶ Can all formulas in  $S$  be transformed in parallel?
- ▶ Why “countable”?

# Transforming sets of formulas

1. Put all formulas in  $S$  into RPF.

Problem in Skolemization step: How do we generate new function symbols if all of them have been used already in  $S$ ?

2. Rename all function symbols in  $S$ :  $f_i^k \mapsto f_{2i}^k$

The result: equisatisfiable countable set  $\{F_0, F_1, \dots\}$ .

Unused symbols: all  $f_{2i+1}^k$

3. Skolemize the  $F_i$  one by one using the  $f_{2i+1}^k$  not used in the Skolemization of  $F_0, \dots, F_{i-1}$

Result is equisatisfiable with initial  $S$ .

# Compactness

## Theorem

*Let  $S$  be a countable set of closed formulas.*

*If every finite subset of  $S$  is satisfiable, then  $S$  is satisfiable.*