Machine Learning on Knowledge Bases

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Introduction

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Machine Learning Algorithms for Premise Selection

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Comparison and Outlook

Introduction

Knowledge Bases and Provers

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- Integration of Automatic Provers in Interactive Provers

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Predict those premises from $\mathcal{P}$ that are likely of use to A for constructing a proof for c

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A proof can be found including the following two lemmas:

$$\text{length}\ xs = \text{length}\ ys \implies \text{zip}(\text{rev}\ xs)(\text{rev}\ ys) = \text{rev}(\text{zip}\ xs\ ys) \quad (1)$$

$$\text{length}(\text{map}\ f\ xs) = \text{length}\ xs \quad (2)$$

Introduction Example

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A straightforward proof uses the following lemmas:

$$\text{used}[\,] = \bigcup_B \text{parts}(\text{initState}\ B) \tag{3}$$

$$X \in \text{parts}(\text{initState}\ B) \Longrightarrow X \in \text{used}\ evs \tag{4}$$

$$(\bigwedge x.x \in A \Longrightarrow x \in B) \Longrightarrow A \subseteq B \tag{5}$$

$$b \in \bigcup_{x \in A} Bx \longleftrightarrow \exists x \in A.b \in Bx \tag{6}$$

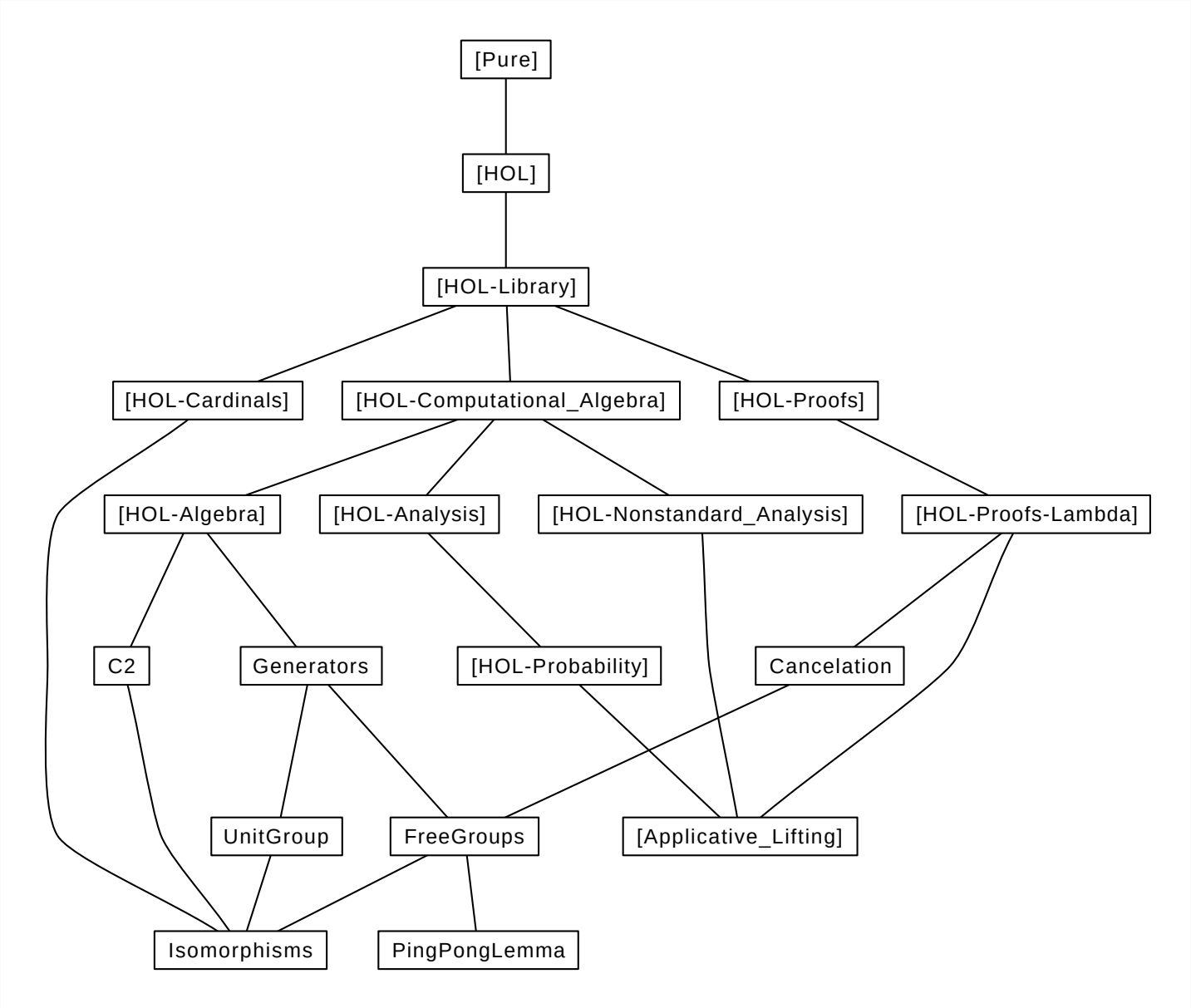
Data Preprocessing

Dependency

Definition (Dependency)

A definition or theorem T depends on some definition, lemma or theorem T', if T' is needed for the proof of T

Theory dependencies graph of Free Groups



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which are simplified to

τ	a	g	$g(h)$
h	$h(\tau)$	$h(a)$	$h(\tau, a)$

Another example of feature extraction

transpose(map(map f) xss) = map(map f)(transpose xss)

has the features:

map
map(fun)
map(map)
map(transpose)
map(map, transpose)
map(list_list)
map(map, list list)
transpose
transpose(map)
transpose(list_list)
fun
list
list_list
List

Stored Information

$$(c, usedPremises(c), F(c))$$

Machine Learning Algorithms for Premise Selection

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$$\text{Relevance}_c(p) =$$

$$\left(\tau_2 \sum_{q \in N \,|\, p \in usedPremises(q)} \frac{n(q,c)}{|usedPremises(q)|} \right) + \begin{cases} n(p,c) & \text{if } p \in N \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

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$$\begin{aligned} & P(\text{p is used in the proof of c'}) \\ \cdot & \prod_{t \in F(c) \cap \bar{F}(p)} P(\text{c' has feature t} \,|\, \text{p is used in the proof of c'}) \\ \cdot & \prod_{t \in F(c) - \bar{F}(p)} P(\text{p has feature t} \,|\, \text{p is not used in the proof of c'}) \\ \cdot & \prod_{t \in \bar{F}(p) - F(c)} P(\text{c' does not have feature t} \,|\, \text{t is used in the proof of c'}) \end{aligned} \tag{11}$$

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$$P(\text{p is used in the proof of c'}) = \frac{r(p)}{K} \tag{12}$$

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$$\text{Relevance}_c(p) = \sigma_1 \ln(r(p)) + \sum_{t \in F(c') \cap \bar{F}(p)} w(f) \ln \frac{\sigma_2 s(p,t)}{r(p)}$$
$$+ \sigma_3 \sum_{t \in \bar{F}(p) - F(c)} w(f) \ln \left(1 - \frac{s(p,t)}{r(t)}\right) + \sigma_4 \sum_{t \in F(c) - \bar{F}(p)} w(f)$$

Premise Selectors in Detail

MePo

For a set K of known symbols let

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1. For each fact, compute as its score $k/(k+u)$
2. Select the facts with the highest score;
   add the features of the selected facts to the set of known symbols K.

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for $p_1, ...p_n$ selected by selection algorithm

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k-Coverage

$$\frac{|\{p_1, ..., p_k\} \cap usedPremises(c)|}{min\{k, |usedPremises(c)|\}}$$

Comparison in Metric

		MaSh		MeSh	
Formalization	MePo	NB	kNN	NB	kNN
Auth	647	104	143	**96**	112
IsaFoR	1332	**513**	604	517	570
Jinja	839	244	306	**229**	256
List	1083	**234**	263	259	271
Nominal2	1045	**220**	276	229	264
Probability	1324	424	422	**393**	395

Figure 1: Average full recall

Comparison in Metric

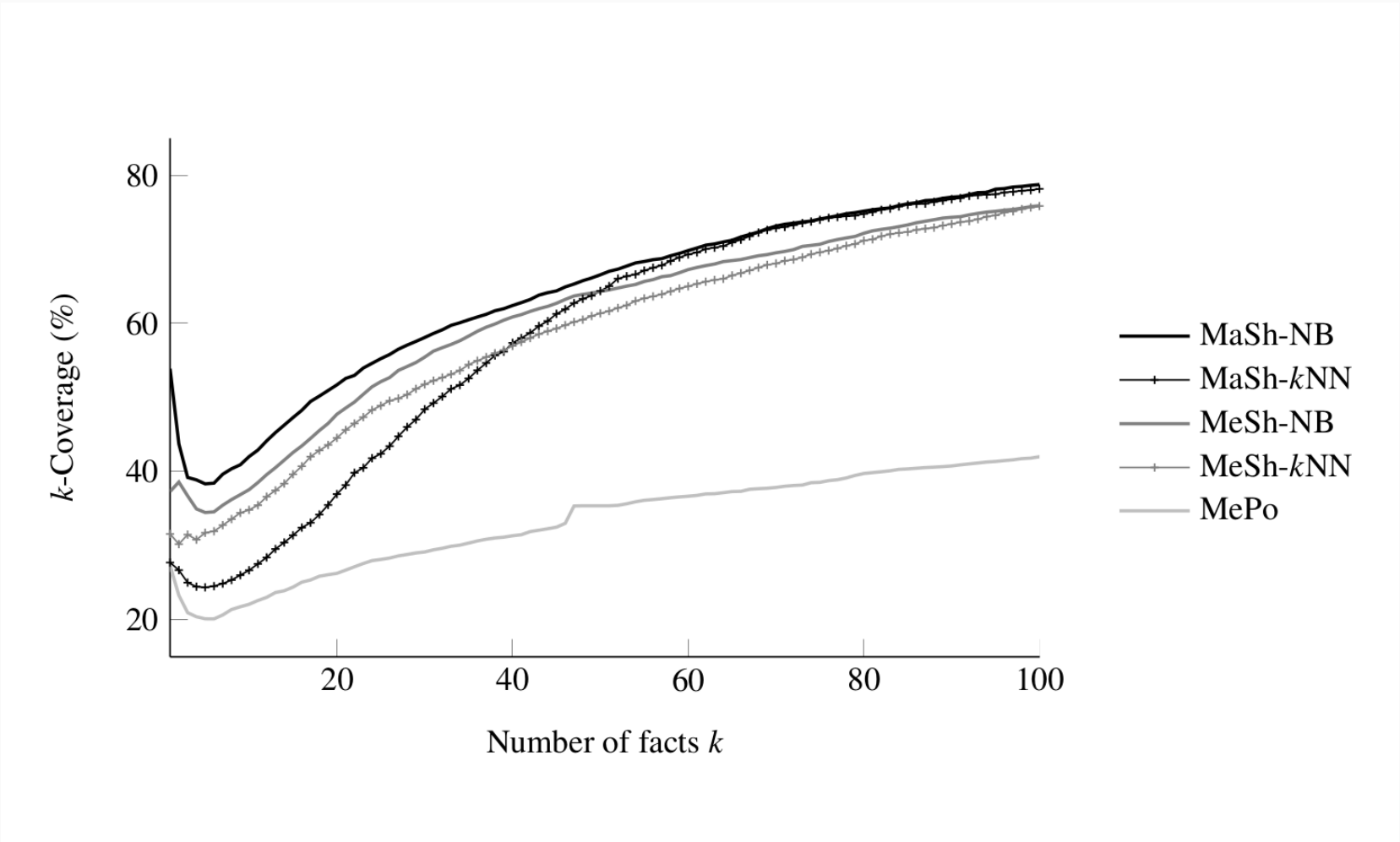

Figure 2: k-Coverage for IsaFoR

Comparison in Performance

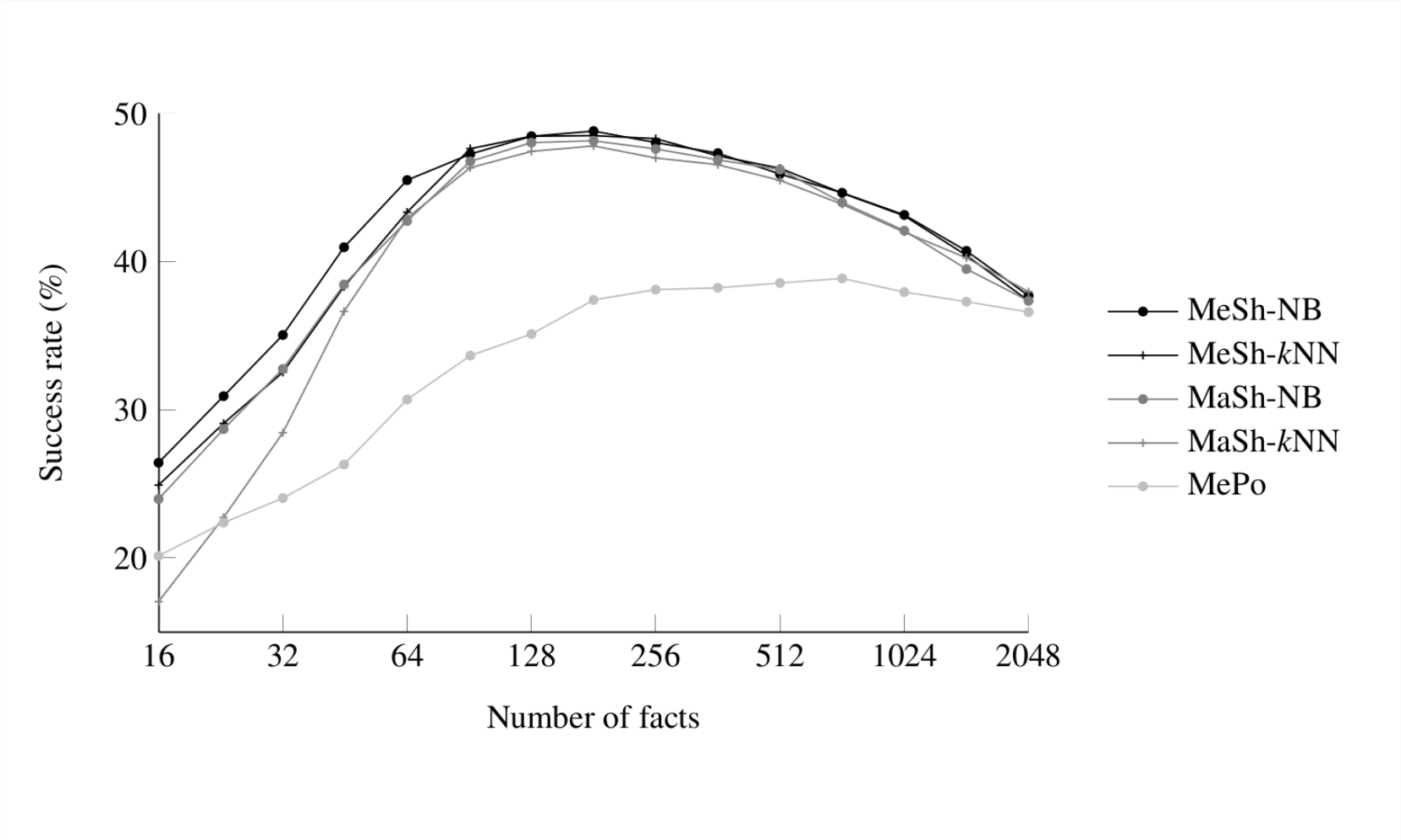

Figure 3: Success rates

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- Fine grained dependency analysis and good feature extraction enable the successful application of machine learning algorithms
- MaSh and MeSh outperform MePo in terms of the chosen metrics
- Similar results for SNoW and MOR compared to Vampire-SInE for MML
- Possibilities of applying other machine learning algorithms, e.g. Neural Networks