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Semantics of Programming Languages

Exercise Sheet 12

Exercise 12.1 AI for Conditionals

Our current constant analysis does not regard conditionals. For example, it cannot figure out, that after executing the program

x:=2; IF x<2 THEN x:=2 ELSE x:=1

x will be constant.

In this exercise we extend our abstract interpreter with a simple analysis of boolean expressions. To this end, modify locale *Val_semilattice* in theory *Abs_Int0.thy* as follows:

- Introduce an abstract domain *'bv* for boolean values, add, analogously to *num'* and *plus'* also functions for the boolean operations and for *less*.
- Modify *Abs_Int0* to accomodate for your changes. Do not modify the locales ending in *_fun*, they are not needed for executable analysis, and can simply be commented out.
- Define a function *bval'* in *Abs_Int1*, and modify the *step'* function to take into account boolean values guaranteed to be false.
- Finally, adapt all theories necessary to get a more precise constant analysis.

Hint: Start with a fresh copy of the IMP/ folder.

Homework 12.1 Complete Lattices

Submission until Tuesday, 27 January 2015, 10:00am.

***Note:* The homework are typical exam exercises. They are not done in Isabelle, so either submit a text file / pdf by email, or hand it in on paper at the beginning of the next tutorial.**

Which of the following ordered sets are complete lattices?

- $\mathbb{N}$, the set of natural numbers $\{0, 1, 2, 3, \ldots\}$ with the usual order
- $\mathbb{N} \cup \{\infty\}$, the set of natural numbers plus infinity, with the usual order and $n < \infty$ for all $n \in \mathbb{N}$.
- A finite set A with a total order $\leq$ on it.

- $\mathbb{B}^*$, the set of all lists of booleans $\{[], [\texttt{True}], [\texttt{False}], \ldots\}$, with prefix order:
  $a \leq b \Longleftrightarrow \exists c.\ b = a@c$.

For each ordered set, prove (no Isabelle proof, only pen & paper) that it is a complete lattice or give a counter example. Note: You do not need to prove the order properties, only the properties to be a complete lattice.

Homework 12.2 Collecting Semantics

Submission until Tuesday, 27 January 2015, 10:00am.

Note: This is a typical exam exercise.

Show the iterative computation of the collecting semantics of the following program in a table like the one on page 228 of the book.

```
x := 0; y := 2 {A0} ;
{A1}
WHILE 0 < y
DO {A2} ( x := x+y; y := y - 1 {A3} )
{A4}
```

Note that two annotations have been suppressed to make the task less tedious. You do not need to show steps where only the suppressed annotations change.

Because the program contains two variables, the state sets in the table should be represented as sets of pairs (x, y). In order to keep the table compact, you can also just write xy, e.g. 02 instead of $(0, 2)$ — the values of the variables do not exceed single digits.

	0								
A_0	{}								
A_1	{}								
A_2	{}								
A_3	{}								
A_4	{}								